Test N°4 Math 232 (to be submitted by Dec 19, 2012)

KFUPM **Semester 121**

Dept. Math. &Stat. **A.Y:2012/2013**

Name: ....

ID: ....

Exercise 1

(a) Exhibit a bijection from $\mathbb{Z}$ to the set of all integers that are 2 more than a multiple of 5.

(b) Write a formula for the inverse of the bijection in part (a).

Exercise 2

Prove the following statements.

(a) $A \times B \approx B \times A$

(b) If $A \approx C$ and $B \approx D$ then $A \times B \approx C \times D$.

(c) $(A \times B) \times C \approx A \times (B \times C)$

(d) If w is any element then $\{w\} \times A \approx A$.

Exercise 3

Define a bijection from the interval $[-2, 8]$ onto the interval $[3, 5]$.

Exercise 4

A study of 115 breakfast eaters shows that 85 also eat lunch, 58 use dental floss regularly, and 27 subscribe to a morning newspaper. Among those who also eat lunch, 52 floss regularly and 15 get the morning paper, and 10 lunch eaters both floss and get the paper. Four flossers neither eat lunch nor get the paper.

(a) How many of those in the study neither eat lunch, nor floss regularly, nor get the morning paper?

(b) How many of those who use dental floss regularly also get the morning paper?

(c) How many of those who get the morning paper neither use dental floss regularly nor eat lunch?

Exercise 5

Show that if $\#A \leq \#B$ then $\#P(A) \leq \#P(B)$.

Exercise 6

(a) Use Cantor's theorem to deduce that $n < 2^n$ for every nonnegative integer n.

(b) Prove that $n < 2^n$ for every nonnegative integer n, using mathematical induction instead of Cantor's theorem.

Exercise 7

Prove or disprove: There exists a bijective function $f:\mathbb{Q}\to\mathbb{R}$.

Prove or disprove: The set $\mathbb{Q}^{100}$ is countably infinite.

Prove or disprove: The set $\mathbb{Z}\times\mathbb{Q}$ is countably infinite.

Prove or disprove: The set $\{0,1\}\times\mathbb{N}$ is countably infinite.

Exercise 8

Prove that if A and B are finite sets with $|A| = |B|$, then any injection $f : A \to B$ is also a surjection. Show this is not necessarily true if A and B are not finite.

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