KFUPM **Semester 121**

Dept. Math. &Stat. **A.Y:2012/2013**

Name: ....

ID: ....

Exercise 1

Label each statement *true* or *false*.

(a) $\{1, 2, 3\} = \{3, 1, 2\}$
(b) $\{1, 2, 3\} = \{2, 1, 3, 3, 2\}$
(c) $\{5, \emptyset\} = \{5\}$
(d) $\{5\} \in \{2, 5\}$
(e) $\emptyset \in \{1, 2\}$

Exercise 2

Give sets A and B such that the statements $A \in B$ and $A \subseteq B$ are both true.

Exercise 3

Let $A = \{1, 2, \pi\}$, and let P be the statement "$x \in A$ and $x \in \mathbb{Z}$." Determine the truth value of each of the following implications, and justify briefly. Take the set $\mathbb{R}$ of real numbers as the universal set.

(a) $(\exists x)P \implies (\forall x)P$
(b) $(\forall x)P \implies (\exists x)P$

Exercise 4

Suppose $A_1 = \{a, b, d, e, g, f\}$, $A_2 = \{a, b, c, d\}$, $A_3 = \{b, d, a\}$ and $A_4 = \{a, b, h\}$

(a) $\bigcup_{i=1}^{4} A_i =$

(b) $\bigcap_{i=1}^{4} A_i =$

Exercise 5

For each $n \in \mathbb{N}$, let $A_n = \{0, 1, 2, 3, \ldots, n\}$.

(a) $\bigcup_{i \in \mathbb{N}} A_i =$

(b) $\bigcap_{i \in \mathbb{N}} A_i =$

Exercise 6

Is $\bigcap_{\alpha \in I} A_\alpha \subseteq \bigcup_{\alpha \in I} A_\alpha$ always true for any collection of sets A_α with index set I?

Exercise 7

If $\bigcap_{\alpha \in I} A_\alpha = \bigcup_{\alpha \in I} A_\alpha$, what do you think can be said about the relationships between the sets A_α?

Exercise 8

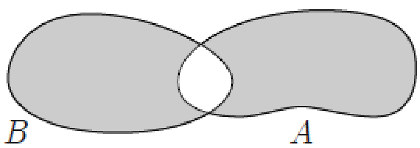
Prove or give a counterexample for each of the following statements.

(1) Let A and B be sets. Then $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$.
(2) Let A and B be sets. Then $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.

Exercise 9

Let A and B be sets. Define the **symmetric difference** to be

$$A \oplus B = (A - B) \cup (B - A)$$



The set $A \oplus B$ consists of those elements of $A \cup B$ that belong to exactly one of the sets A, B. Prove the following statements. (Assume only that A and B are sets. Do not assume that we know what their elements are.)

(a) $A \oplus A = \emptyset$

(b) $A \oplus \emptyset = A$

(c) $(A \oplus B) - (A - B) = B - A$

(d) $A \oplus B = \emptyset \Longrightarrow A = B$ (Therefore, combining parts (a) and (d) gives the equivalence $A \oplus B = \emptyset \Longleftrightarrow A = B$.)

(e) $(A \oplus B) \oplus C = A \oplus (B \oplus C)$ (Start with a Venn diagram.)

Test N°2 Math 232 (November 22, 2012)

Exercise 10

Determine each of the following sets.

(a) $P(\{2\})$

(b) $P(P(\{2\}))$

(c) $P(P(P(\{2\})))$

Exercise 11

(a) Show that if A and B are sets then $\emptyset \notin P(A) - P(B)$.

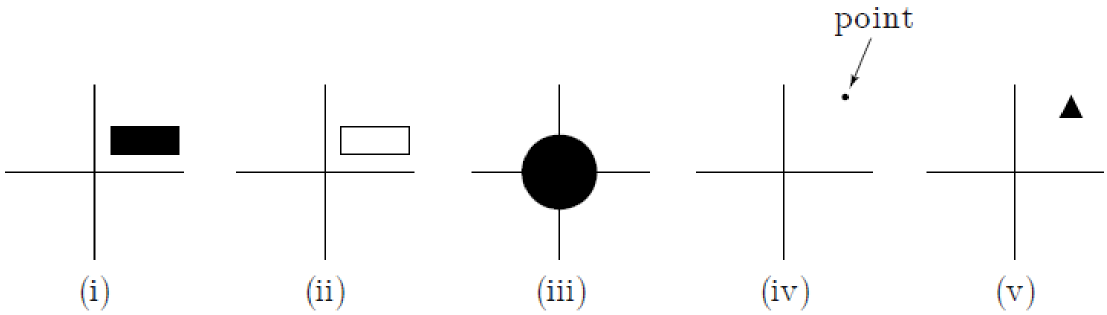
(b) Deduce from part (a) that the difference of two power sets can never be a power set.

(c) Prove that $P(A - B) \subseteq (P(A) - P(B)) \cup \{\emptyset\}$.

Exercise 12

(a) Exhibit a subset S of $\{1,2,3\} \times \{1,2,3\}$ for which there are no subsets A and B of $\{1,2,3\}$ such that $A \times B = S$.

(b) Which of the following pictures represent(s) a set that has the form $A \times B$ for some $A,\ B \subseteq \mathbb{R}$?



Exercise 13

List all the partitions of the set $\{1,2,3,4,5\}$ that have no one-element blocks.

Exercise 14

Let $S = \{1, 2, 3\}$. In each case give an example of a relation R on S that has the stated properties.

(a) R is not symmetric, not reflexive, and not transitive.

(b) R is transitive and reflexive, but not symmetric.

Exercise 15

Prove for every integer $n \geq 1$:

$$1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$