KFUPM **Semester 121**

Dept. Math. &Stat. **A.Y:2012/2013**

Test N°1 Math 232

(October 01, 2012)

Name: .....

ID: .....

(I)

A *statement* is a declarative sentence

Decide whether or not the following are statements. In the case of a statement, say if it is true or false, if possible.

- Every real number is an even integer.
- If x and y are real numbers and $5x = 5y$, then $x = y$.
- The integer x is a multiple of 7.
- Either x is a multiple of 7, or it is not.
- In the beginning, God created the heaven and the earth.

(II)

Without changing their meanings, convert each of the following sentences into a sentence having the form "*If P, then Q*."

- A matrix is invertible provided that its determinant is not zero.
- Whenever people agree with me I feel I must be wrong. (Oscar Wilde)

(III)

Without changing their meanings, convert each of the following sentences into a sentence having the form "*P if and only if Q*."

- For matrix A to be invertible, it is necessary and sufficient that $\det(A) \neq 0$.
- If $xy = 0$ then $x = 0$ or $y = 0$, and conversely.
- For an occurrence to become an adventure, it is necessary and sufficient for one to recount it. (Jean-Paul Sartre)

(IV)

Write a truth table for four among the following nine statements:

1. $P \vee (Q \Rightarrow R)$
2. $(Q \vee R) \Leftrightarrow (R \wedge Q)$
3. $\sim (P \Rightarrow Q)$
4. $\sim (P \vee Q) \vee (\sim P)$
5. $(P \wedge \sim P) \vee Q$
6. $(P \wedge \sim P) \wedge Q$
7. $(P \wedge \sim P) \Rightarrow Q$
8. $P \vee (Q \wedge \sim R)$
9. $\sim (\sim P \vee \sim Q)$

(V)

Suppose the statement $((P \wedge Q) \vee R) \Rightarrow (R \vee S)$ is false. Find the truth values of P, Q, R and S. (This can be done without a truth table.)

(VI)

Suppose P is false and that the statement $(R \Rightarrow S) \Leftrightarrow (P \wedge Q)$ is true. Find the truth values of R and S. (This can be done without a truth table.)

(VII)

Decide whether or not the following pairs of statements are logically equivalent.

- $P \wedge Q$ and $\sim(\sim P \vee \sim Q)$

- $\sim(P \Rightarrow Q)$ and $P \wedge \sim Q$

- $(P \Rightarrow Q) \vee R$ and $\sim((P \wedge \sim Q) \wedge \sim R)$

- $P \vee (Q \wedge R)$ and $(P \vee Q) \wedge R$

(VIII)

Using "proof by contradiction", show that:

If $a, b \in \mathbb{Z}$ and $a \geq 2$, then $a \nmid b$ or $a \nmid (b+1)$.