King Fahd Univ. Petr. & Min.   **Term 121**

Depart. Math. and Stat.   **A.Y:2012/013**

Final Exam Math 232 (January 02, 2013)

07:00-10:00 pm

Name: ....

ID: ....

Exercise 1: Show that

$$1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + 4 \cdot 6 + \cdots + n(n+2) = \frac{n(n+1)(2n+7)}{6}.$$

Solution.

Exercise 2:

Prove that $\sqrt[3]{2}$ is irrational.

Solution.

Exercise 3:

Describe the partition of $\mathbb{Z}$ resulting from the equivalence relation $\equiv$ (mod 4).

Solution.

Exercise 4:

A study of 115 breakfast eaters shows that 85 also eat lunch, 58 use dental floss regularly, and 27 subscribe to a morning newspaper. Among those who also eat lunch, 52 floss regularly and 15 get the morning paper, and 10 lunch eaters both floss and get the paper. Four flossers neither eat lunch nor get the paper.

(a) How many of those in the study neither eat lunch, nor floss regularly, nor get the morning paper?

(b) How many of those who use dental floss regularly also get the morning paper?

(c) How many of those who get the morning paper neither use dental floss regularly nor eat lunch?

Solution.

Exercise 5:

Let A and B be sets. Prove that $A \sim B$ implies that $\mathcal{P}(A) \sim \mathcal{P}(B)$.

Solution.

Exercise 6:

Show that the integers that are 3 more than a multiple of 7 constitute a countable set.

Solution.

Exercise 7:

Prove that if n is an odd integer then $n^2 \equiv 1 \pmod{8}$.

Solution.

Exercise 8:

Show that if $m \mid n$ then $\varphi(m) \mid \varphi(n)$.

Solution.

Exercise 9:

Find the order of each element of the group

(a) S_3.

(b) $(\mathbb{Z}_7, +)$.

(c) $(\mathbb{Z}_8, +)$.

(d) $(\mathbb{Z}_7 - \{0\}, \cdot)$.

Solution.

Exercise 10:

Suppose G is a group of prime order p, and $a \in G$. What are the possible orders of a? If a is not the identity, what is the order of a? How many elements of G are in the subgroup of G generated by a? How many elements of G are generators of the cyclic group G?

Solution.