Key

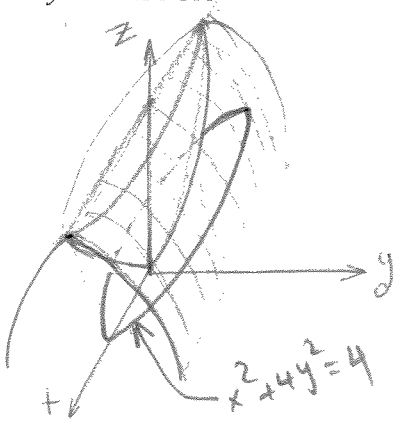
Serial No.:_____ Student Name:_____ Student Number:_____

Instructor: M. Z. Abu-Sbeih Math 201.11 – Q6 Date: 26-12-2012

Show all your work. No credits for answers not supported by work.

Rectangular: (17 Points) Use triple integral to find the volume of the solid bounded above by the parabolic cylinder $z = 4 - y^2$ and below by the elliptic paraboloid $z = x^2 + 3y^2$. SKETCH THE REGION.

$$V = \int_{-2}^{2} \int_{-\frac{1}{2}\sqrt{4-x^2}}^{\frac{1}{2}\sqrt{4-x^2}} \int_{x^2+3y^2}^{4-y^2} dz\,dy\,dx.$$

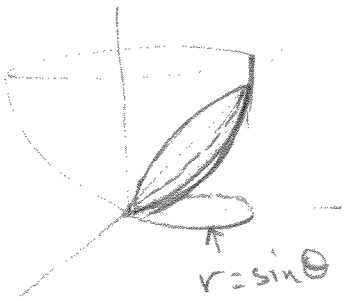


Cylindrical: (18 Points) Use triple integral to find the volume of the solid bounded above by the plane $z = y$ and below by the paraboloid $z = x^2 + y^2$. SKETCH THE REGION.

Intersection: $y = x^2 + y^2$; $r\sin\theta = r^2$

$r = \sin\theta$.

$$\therefore V = \int_{0}^{\pi} \int_{0}^{\sin\theta} \int_{r^2}^{r\sin\theta} r\,dz\,dr\,d\theta$$



Spherical: (25 Points) Use triple integral to find the volume of the solid bounded above by the cone $z^2 = x^2 + y^2$, below by the xy-plane, and on the side by the hemisphere $z = \sqrt{4 - x^2 - y^2}$. SKETCH THE REGION.

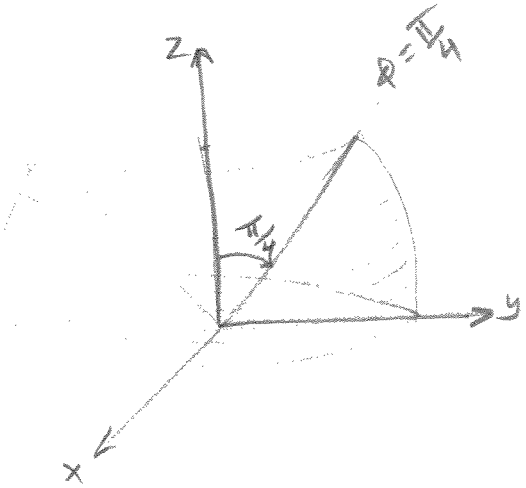
$$V = \int_{0}^{2\pi} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_{0}^{2} \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta$$

$$= \left(\int_{0}^{2\pi} d\theta\right)\left(\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin\phi\,d\phi\right)\left(\int_{0}^{2} \rho^2\,d\rho\right)$$

$$= (2\pi)\left(-\cos\phi\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}\right)\left(\frac{8}{3}\right)$$

$$= \frac{16\pi}{3}\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{8\sqrt{2}\,\pi}{3}$$



$0 \le \theta \le 2\pi$

$\frac{\pi}{4} \le \phi \le \frac{\pi}{2}$

$0 \le \rho \le 2$