Key

Serial No.:______ Student Name:______________________ Student Number: ______

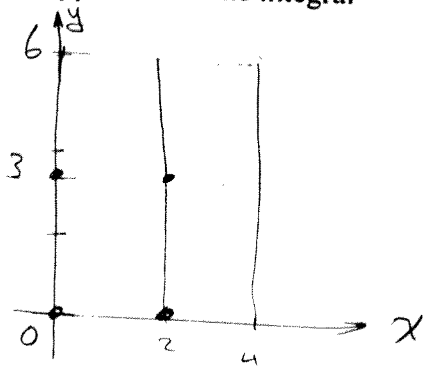
Instructor: M. Z. Abu-Sbeih   Math 201.11 – Q5   Date: 10-12-2012

Show all your work. No credits for answers not supported by work.

Problem 1: (15 Points) Use the lower left end point with $n = m = 2$ to approximate the integral

$$\iint_R (x^2 - y)\,dA,$$

where $R = [0,4] \times [0,6] = \{(x, y): 0 \le x \le 4, 0 \le y \le 6\}$



4 points: $(0,0), (0,3), (2,0), (2,3)$.

2) $\Delta A = 2 \times 3 = 6.$

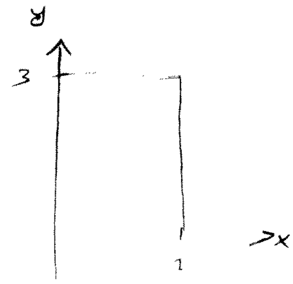
$f(x,y) = x^2 - y.$

6 $$\iint_R (x^2 - y)\,dA \cong \Delta A\left[f(0,0) + f(0,3) + f(2,0) + f(2,3)\right]$$

$$= 6\left[0 + (0-3) + (4-0) + (4-3)\right]$$

3 $$= 6(2) = 12$$

Problem 2: (15 Points) Find the volume of the solid bounded by the surface $z = f(x, y) = x^2 e^y$ and the planes $z = 0,\ x = 0,\ x = 2,\ y = 0,$ and $y = 3$.



$$V = \iint_D x^2 e^y\,dA.$$

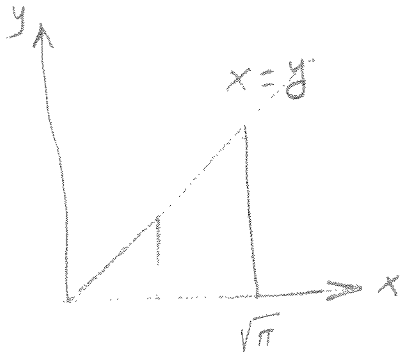
5) $$= \int_0^2 \int_0^3 x^2 e^y\,dy\,dx$$

5+5. $$= \left(\int_0^2 x^2\,dx\right)\left(\int_0^3 e^y\,dy\right)$$

$$= \left(\frac{x^3}{3}\Big|_0^2\right)\left(e^y\Big|_0^3\right) = \left(\frac{8}{3}\right)\left(e^3 - 1\right)$$

Problem 3: (15 Points) Evaluate

$$\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} \sin x^2 \, dxdy$$



$$= \int_0^{\sqrt{\pi}} \int_0^{x} \sin x^2 \, dy \, dx$$

$x = y$

$x = \sqrt{\pi}$

$$= \int_0^{\sqrt{\pi}} y \sin x^2 \Big|_{y=0}^{y=x} dx$$

$$= \frac{1}{2} \int_0^{\sqrt{\pi}} 2x \sin x^2 \, dx$$

$$= -\frac{1}{2} \cos x^2 \Big|_0^{\sqrt{\pi}} = -\frac{1}{2} \left[\cos \pi - \cos 0 \right]$$

$$= -\frac{1}{2} \left[-1 - 1 \right] = 1 .$$

Problem 4: (15 Points) Use double integral to calculate the area of the region enclosed by $y = x^2$ and $x + y = 2$.

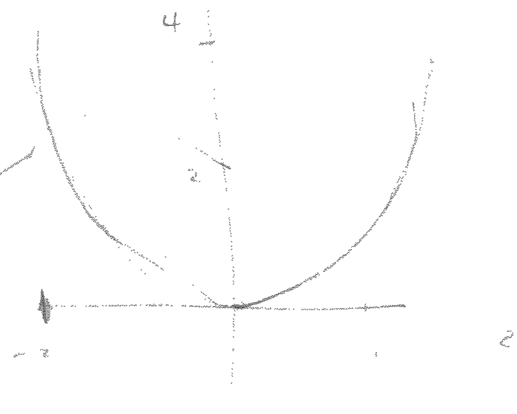
Intersection $x^2 = 2 - x \rightarrow x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2, 1$

$\rightarrow -2 \le x \le 1$;

$x^2 \le y \le 2 - x$



$$\iint_R dA = \int_{-2}^{1} \int_{x^2}^{2-x} dy \, dx$$

$$= \int_{-2}^{1} (2 - x - x^2) \, dx$$

$$= \left[2x - \frac{x^2}{2} - \frac{1}{3} x^3 \right]_{-2}^{1}$$

$$= \frac{9}{2}$$

May use:

$$\int_0^1 \int_{-\sqrt{y}}^{\sqrt{y}} dx\,dy + \int_1^4 \int_{-\sqrt{y}}^{2-y} dx\,dy$$