Serial No.:_______ Student Name:__________________ Student Number: _______

Instructor: M. Z. Abu-Sbeih Math 201.11 – Q1 Date: 19-9-2012

Show all your work. No credits for answers not supported by work.

Write neatly and eligibly. You may loose credits for messy work.

Problem 1: (18 Points:4,5,4,5) Consider the curve with parametric equations

$x = \sin t,\ y = \cos 2t,\ 0 \le t \le \frac{\pi}{2}$.

a) Find the rectangular equation of the curve.

(2 pts) $y = \cos 2t = 1 - 2\sin^2 t$

(2 pts) $= 1 - 2x^2$

b) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

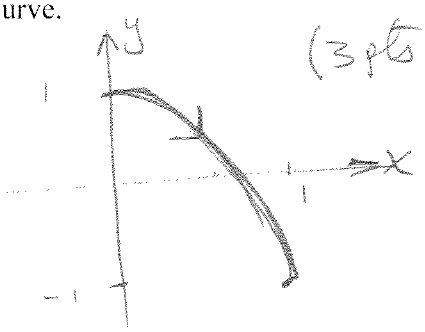
(3 pts) $y' = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2\sin 2t}{\cos t} = \frac{-4\sin t\cos t}{\cos t} = -4\sin t$

(2 pts) $\frac{d^2y}{dx^2} = \frac{dy'/dt}{dx/dt} = \frac{-4\cos t}{\cos t} = -4$

c) Sketch the graph and clearly indicate the orientation of the curve.

(1 pt)

t	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
x	0	$\frac{1}{\sqrt{2}}$	1
y	1	0	-1



(3 pts)

d) Write an integral equation in parametric which gives the length of the curve. (Simplify but DO NOT EVALUATE THE INTEGRAL)

2 pts $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \cos^2 t + 4\sin^2 2t$

(1 pt) $L = \int_0^{\pi/2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt =$

(1 pt) $= \int_0^{\pi/2} \sqrt{\cos^2 t + 4\sin^2 2t}\, dt$

(1 pt) $= \int_0^{\pi/2} \cos t\sqrt{1 + 16\sin^2 t}\, dt$

Problem 2: **(17 points:4,5,4,4)** Consider the polar curve $r = 1 - 2\sin\theta$.

a. Find the rectangular equation of the curve.

(2 pts) $r^2 = r - 2r\sin\theta$

(2 pts) $x^2 + y^2 = \sqrt{x^2+y^2} - 2y$

$x^2 + y^2 + 2y = \sqrt{x^2+y^2}$

b. Find the slope of the tangent line when $\theta = 0$.

(1 pt) (1 pt)

$\boxed{\theta = 0 \Rightarrow r = 1 \;\Big|\; \frac{dr}{d\theta} = -2\cos\theta}$

(1 pt) $$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

(2 pts) $$= \left.\frac{-2\cos\theta\sin\theta + r\cos\theta}{-2\cos\theta\cos\theta - r\sin\theta}\right|_{\substack{\theta=0\\ r=1}} = \frac{0+1}{-2-0} = -\frac{1}{2}.$$

c. Find all symmetries of the curve. (Show why)

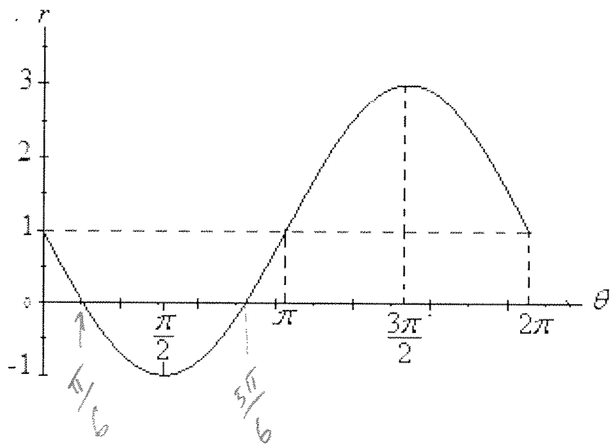
(1 pt) With respect to polar axis: No

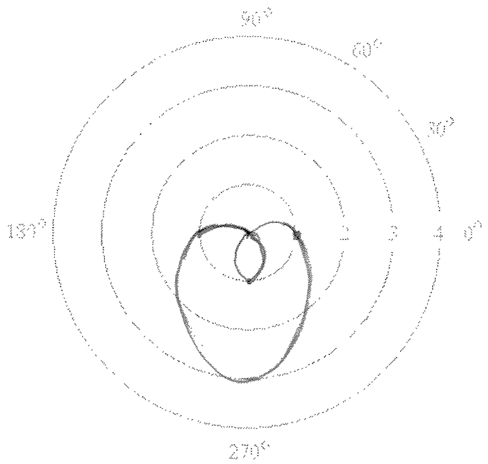
(2 pts) With respect to line $\theta = \frac{\pi}{2}$: yes Replace θ by $(\pi - \theta)$ we get.

$1 - 2\sin(\pi - \theta) = 1 - 2\sin\theta = r.$

(1 pt) With respect to the pole: No

d. Use the graph of the equation $r = 1 - 2\sin\theta$ given in Cartesian plane (to the left) to sketch the polar curve.

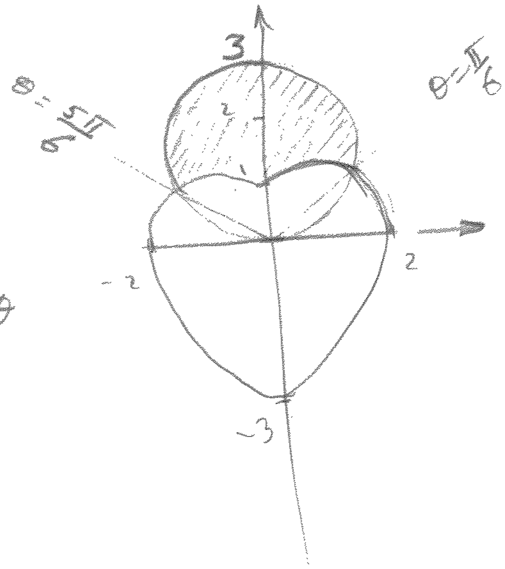




Problem 3: **(10 points)** Write an integral equation in polar which gives the area inside $r = 3\sin\theta$ and outside $r = 2 - \sin\theta$. (Simplify but DO NOT EVALUATE THE INTEGRAL)

4 pts $\quad 3\sin\theta = 2 - \sin\theta$

$\sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6},\ \frac{5\pi}{6}$



(6 pts) $\quad A = \frac{1}{2}\int_{\pi/6}^{5\pi/6} \left[(3\sin\theta)^2 - (2-\sin\theta)^2\right] d\theta$

Problem 4: **(15 points:10,5)** consider the curve $r = 1 + e^{\theta/2}$. $\quad 0 \le \theta \le \pi$

a. Write an integral equation in polar which gives the length of the curve. (DO NOT EVALUATE THE INTEGRAL)

(2 pts) $\quad \frac{dr}{d\theta} = \frac{1}{2} e^{\theta/2}$

(2 pts) $\quad r^2 + \left(\frac{dr}{d\theta}\right)^2 = \left(1 + e^{\theta/2}\right)^2 + \frac{1}{4} e^{\theta}$

(6 pts) $\quad L = \int_0^{\pi} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta = \int_0^{\pi} \sqrt{\left(1 + e^{\theta/2}\right)^2 + \frac{1}{4} e^{\theta}}\, d\theta$

b. Write an integral equation in polar which gives the surface area of the solid generated by rotating the curve about the polar axis. (DO NOT EVALUATE THE INTEGRAL)

(2 pts) $\quad S = \int_0^{\pi} 2\pi y \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

(2 pts) $\quad = \int_0^{\pi} 2\pi\, r\sin\theta \sqrt{\left(1 + e^{\theta/2}\right)^2 + \frac{1}{4} e^{\theta}}\, d\theta.$

(1 pt) $\quad = \int_0^{\pi} 2\pi \left(1 + e^{\theta/2}\right)\sin\theta \sqrt{\left(1 + e^{\theta/2}\right)^2 + \frac{1}{4} e^{\theta}}\, d\theta$