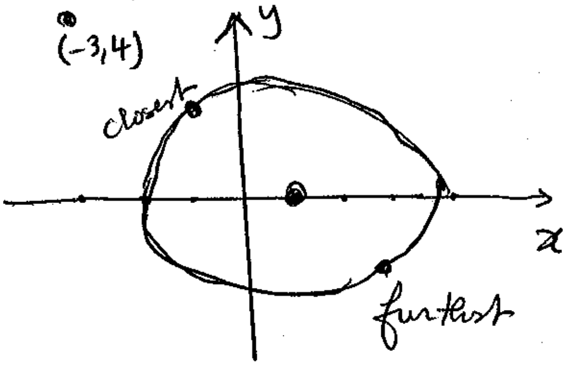
Q.2 (5 points): Use the Lagrange multiplier method to find the closest and furthest points of the circle $(x-1)^2 + y^2 = 8$ to the point $(-3, 4)$

We minimize and maximize $d^2 = (x+3)^2 + (y-4)^2$
subject to $(x-1)^2 + y^2 - 8 = 0$



$f(x,y) = (x+3)^2 + (y-4)^2$

$\nabla f = \lambda \nabla g \iff \langle 2(x+3), 2(y-4) \rangle = \lambda \langle 2(x-1), 2y \rangle$

$\iff \begin{cases} \lambda(x-1) = x+3 = x-1+4 \\ \lambda y = (y-4) \end{cases}$

$\iff \begin{cases} (\lambda-1)(x-1) = 4 \\ (\lambda-1) y = -4 \end{cases}$

$\implies (\lambda-1)^2 \left[(x-1)^2 + y^2 \right] = 32$

$8(\lambda-1)^2 = 32 \implies (\lambda-1)^2 = 4$

$\implies \lambda = -1, 3$

$\boxed{\lambda = -1}$ gives $(-1, 2) \implies f(-1,2) = 4 + 4 = 8$

$\boxed{\lambda = 3}$ gives $(3, -2) \implies f(3,-2) = 36 + 36 = 72$

So the closest point ~~on~~ the circle is $(-1, 2)$
and the furthest point on ... is $(3, -2)$

Math 201

Quiz 5

10/ 12/ 2012

Name: **ID #**

Q.1 (5 points): Find all critical points of $f(x,y) = 2x^2 + 3y^2 - 6x^2y - 2$ and determine the *max*, *min* or saddle point.

Sol. $f_x = 4x - 12xy = 0 \iff 4x(1-3y) = 0$

$f_y = 6y - 6x^2 = 0 \iff y = x^2$ $\Rightarrow$

$(0,0), (-\frac{1}{\sqrt{3}}, \frac{1}{3}), (\frac{1}{\sqrt{3}}, \frac{1}{3})$ are the critical points

$f_{xx} = 4 - 12y$, $f_{xy} = -12x$, $f_{yy} = 6$

$D = f_{xx} f_{yy} - (f_{xy})^2 = 24 - 72y - 144x^2$

- $(0,0)$: $D(0,0) = 24 > 0$, $f_{xx}(0,0) = 4 > 0$

  $\therefore (0,0)$ local minimum.

- $D(-\frac{1}{\sqrt{3}}, \frac{1}{3}) = 24 - 24 - \frac{144}{3} = -48 < 0$

  $(-\frac{1}{\sqrt{3}}, \frac{1}{3})$ saddle point

- $D(\frac{1}{\sqrt{3}}, \frac{1}{3}) = 24 - 24 - 48 < 0$

  $(\frac{1}{\sqrt{3}}, \frac{1}{3})$ saddle point