Math 201

Quiz 1

19/ 9/ 2012

Name: ID #

Problem 1 (5 points): Sketch the curve given by the parametric equations

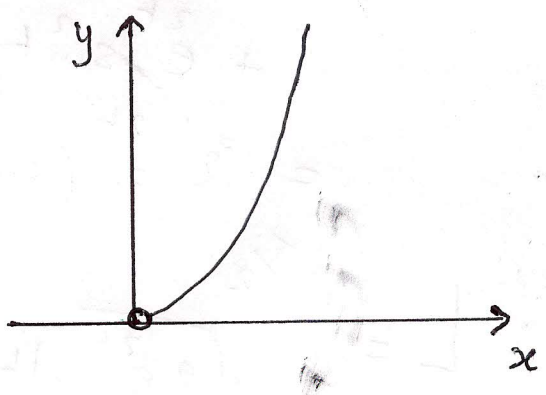
$$x = e^t \qquad y = e^{2t}$$

and say what it represents.

t	$-\infty$	-2	-1	0	1	∞
x	0	$1/e^2$	$1/e$	1	e	∞
y	0	$1/e^4$	$1/e^2$	1	e^2	∞

This is a parabola

$y = x^2$, $x > 0$



Problem 2 (5 points):
Find the length of the curve

$$C = \left\{ (e^t \cos t, e^t \sin t), \qquad 0 \le t \le \frac{\pi}{2} \right\}$$

$$L = \int_0^{\pi/2} \sqrt{x'^2(t) + y'^2(t)}\, dt$$

$$x(t) = e^t \cos t, \quad y(t) = e^t \sin t$$

$$x'(t) = e^t \cos t - e^t \sin t$$

$$y'(t) = e^t \sin t + e^t \cos t$$

$$x'^2(t) + y'^2(t) = e^{2t}\cos^2 t + e^{2t}\sin^2 t - 2e^{2t}\cos t \sin t + e^{2t}\sin^2 t + e^{2t}\cos^2 t + 2e^{2t}\sin t \cos t$$

$$= 2e^{2t}(\cos^2 t + \sin^2 t) = 2e^{2t}.$$

So $$L = \int_0^{\pi/2} \sqrt{2e^{2t}}\, dt = \sqrt{2} \int_0^{\pi/2} e^t\, dt = \sqrt{2}\, e^t \Big|_0^{\pi/2}$$

$$= \sqrt{2}\left(e^{\pi/2} - 1\right).$$