## King Fahd University of Petroleum and Minerals Mathematical Sciences Department MATH 131 Final Exam Duration: 3h Justify all your answers. A good presentation is a must!

 Ia) [12pts] Solve the following LP problem using the simplex method

$$\begin{array}{rll} \max & z & : & = 2x + 3y \\ \text{subject to} & \begin{cases} x + 2y \leq 6 \\ 2x + y \leq 7 \\ x \geq 0, y \geq 0 \end{cases} \end{array}$$

Specify all the steps (entering variables, departing variables, pivots,...)

**Ib)** [4pts] For the Linear Programming Problem in (Ia), draw the feasible region, identify the vertices and the vertex giving the optimal solution.

**II)** [4pts] Express the shaded region in terms of A, B, C and D and their complements.



**IIIa)** [4pts] Find the coefficient of  $y^5$  in the binomial expansion of  $(x + 2y)^{10}$ 

IIIb) A die is tossed 5 times and the sequence of numbers is recorded.i)[3pts] Use the multiplication principle todetermine how many sequences are possible.

ii) [3pts] How many sequences have at least 2 'four'?

**IVa)** A pair of dice is tossed and the numbers on the top faces recorded. One of the dice is fair while the other has two faces showing a 2, one face showing a 3, two faces showing a 4, and one face showing a 5.

i) [8pts] Find the sample space and the probability model of the experiment.

ii) [3pts] Compute the probability of the events that at least one of the dice shows a 4

iii) [3pts] Compute the probability of the events that the sum of the numbers on the top faces of the dice is 8

vi) [3pts] Compute the probability of the events that neither a 2 nor a 4 appears

**v)** [3pts] Compute the probability of the events that the difference between the two numbers is 2.

**IVb)** Let  $P(E) = \frac{1}{5}$  and  $P(E \cup F) = \frac{1}{3}$ . **i)** [3pts] If E and F are independent events, compute P(F).

ii) [3pts] If E and F are mutually exclusive, compute P(F).

**IVc)** A random variable Y has the probability distribution shown below,

Value of $Y$	0	1	2	3	4
Probability	0.02	0.18	0.3	0.2	0.3

i) [4pts] Find the expectation of Y.

ii) [4pts] Find the variance of Y.

iii) [3pts] Find the standard deviation of Y.

 $\mathbf{iv}$  [4pts] Draw the histogram of the probability function.

**IVd)** Let X be a random variable with mean 70 and variance 16. **i)** [4pts] Estimate the probability  $P(62 \le X \le 78)$ 

ii) [4pts] For what value of k,  $P(70 - k \le X \le 70 + k)$  is at least 0.90

Va) Find the effective rate of interest equivalent to 8% compounded i)[4pts] semi-annually

ii)[4pts] continuously

**Vb)** Paul borrows \$ 12000 on January 01, 1996 at an interest rate of 12% compounded monthly.

i) [4pts] What amount must be paid back on July 01, 1998?

ii) [4pts] What is the interest on the loan?

**Vc)** [5pts] Find the monthly payment if \$10000 is amortized for 4 years at 9% compounded semiannually.