King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 102
Final Exam
Term 121
Wednesday , January 09, 2013
Net Time Allowed: 180 minutes

MASTER VERSION

1. $\displaystyle\int \frac{8x^2}{(1+x)^2-(1-x)^2}dx =$

(a) $x^2 + C$

(b) $4x + C$

(c) $\ln|x| + C$

(d) $2\ln|x| + C$

(e) $\frac{1}{3}x^3 + C$

2. If $f(x) = \begin{cases} 1 & -5 \le x \le 0 \\ x & 0 < x \le 2 \\ 3x^2 & 2 \le x \le 6 \end{cases}$

Then $\int_{-1}^{3} f(x)\,dx =$

(a) 22

(b) 21

(c) 20

(d) 23

(e) 24

3. If $F(x) = \int_1^{2x-1} \sin\left(\frac{\pi}{2}t^2\right)\, dt$ and $G(x) = xF(x)$, then $G'(1) =$

(a) 2

(b) $2\,\pi$

(c) π

(d) 3

(e) 0

4. If the first few terms of the Taylor series of $f(x) = \cos(\pi x)$ about $a = 1$ are given by

$$c_0 + c_1(x-1) + c_2(x-1)^2 + c_3(x-1)^3 + \ldots$$

then $c_0 + c_1 + c_2 + c_3 =$

(a) $\dfrac{\pi^2}{2} - 1$

(b) 0

(c) $\pi^2 - 1$

(d) $2\pi + 1$

(e) $-1 + \pi - \pi^2 + \pi^3$

5. The series $\sum_{n=1}^{\infty} \frac{2n}{n+1}$ is

(a) divergent

(b) absolutely convergent

(c) convergent and its sum is 2

(d) conditionally convergent

(e) neither convergent nor divergent

6. The **length** of the curve $y = \frac{2}{3}x^{3/2} + 1$, $0 \leq x \leq 1$, is equal to

(a) $\frac{2}{3}(\sqrt{8} - 1)$

(b) $\frac{2}{3}$

(c) $\frac{4}{3}\sqrt{2}$

(d) $\sqrt{8} + \frac{3}{2}$

(e) $\sqrt{8} - \frac{2}{3}$

7. $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin t\, \cos t}{\sqrt{1+\cos^2 t}}\, dt =$

(a) $\sqrt{2}-1$

(b) 0

(c) $\dfrac{1}{2}$

(d) $\dfrac{\sqrt{2}}{2}$

(e) 2

8. $\displaystyle\int_0^{\sqrt{2}} \frac{x^{11}}{4+x^4}\, dx =$

(a) $-2+4\ln 2$

(b) $3+4\ln 2$

(c) $-1+2\ln 2$

(d) $2+2\ln 2$

(e) $1+2\ln 2$

9. Let R be the region lying in the **first quadrant** and bounded by the curves $y = \sqrt[3]{x}$ and $y = x^3$. The **volume** of the solid generated by rotating R about the x-axis is equal to

(a) $\frac{16\pi}{35}$

(b) $\frac{\pi}{35}$

(c) $16\,\pi$

(d) π

(e) $\frac{5\,\pi}{7}$

10. The **volume** of the solid generated by rotating the region enclosed by the curves

$$y = \frac{1}{x},\ y = 0,\ x = 1,\ x = 2$$

about the line $x = 5$ is given by

(a) $2\pi \int_1^2 \left(\frac{5}{x} - 1\right) dx$

(b) $2\pi \int_1^2 \left(1 - \frac{5}{x}\right) dx$

(c) $2\pi \int_1^2 \left(1 + \frac{5}{x}\right) dx$

(d) $2\pi \int_1^5 \left(\frac{3}{x} - 1\right) dx$

(e) $2\pi \int_1^2 \left(\frac{1}{x} - 5\right) dx$

11. The **area** of the region bounded by the curves

$$y = \sqrt{x},\ y = \frac{1}{2}x,\ x = 9$$

is equal to

(a) $\frac{59}{12}$

(b) $\frac{23}{12}$

(c) $\frac{11}{12}$

(d) $\frac{31}{12}$

(e) $\frac{17}{12}$

12. If the sequence of partial sums of the series $\sum_{n=1}^{\infty} a_n$ is given by $S_n = \frac{e^n}{n},\ n \geq 1,$ then the series $\sum_{n=1}^{\infty} a_n$ is

(a) divergent

(b) convergent and its sum is 0

(c) convergent and its sum is e

(d) convergent and its sum is 1

(e) convergent, but the sum cannot be found.

13. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[7]{n}}$ is

(a) conditionally convergent

(b) absolutely convergent

(c) divergent

(d) neither convergent nor divergent

(e) a convergent $p-$ series

14. $\int x \tan^{-1} x \, dx =$

(a) $\frac{1}{2}\left(x^2 \tan^{-1} x + \tan^{-1} x - x\right) + C$

(b) $x^2 \tan^{-1} x - \tan^{-1} x + C$

(c) $\frac{1}{2}\left(x \tan^{-1} x - x^2\right) + C$

(d) $\frac{x}{x^2+1} + \tan^{-1} x + C$

(e) $\frac{1}{2} x^2 \tan^{-1} x - \tan^{-1} x + C$

15. By applying **the ratio test** to the series

$$\sum_{n=0}^{\infty} \frac{\sqrt{1+n}}{1+(1+n)^2},$$

we conclude that

(a) the test is inconclusive

(b) the series is convergent

(c) the series is divergent

(d) the series is absolutely convergent

(e) the series is conditionally convergent.

16. The improper integral $\int_0^1 \frac{1}{2-3x}\,dx$ is

(a) divergent

(b) convergent and its value is $-\ln 2$

(c) convergent and its value is 0

(d) convergent and its value is $\ln 5$

(e) convergent and its value is 3

17. The sequence $\left\{\frac{2-\sin n}{3^n}\right\}_{n=1}^{\infty}$

(a) converges to 0

(b) converges to 2

(c) converges to $\frac{2}{3}$

(d) converges to 1

(e) is divergent

18. $\int \frac{\csc^2 t}{\cot^2 t + \cot t}\, dt =$

(a) $\ln|1+\tan t| + C$

(b) $\ln|\cot^2 t + \cot t| + C$

(c) $\ln|1+\cot t| + C$

(d) $\cos t + \sin(2t) + C$

(e) $\sin t + \cot t + C$

19. If the curve $y = e^{2x}, 0 \leq x \leq 1$, is rotated about the y-axis, then the **surface area** of the generated surface is given by

(a) $\int_0^1 2\pi x\sqrt{1+4e^{4x}}\,dx$

(b) $\int_0^1 2\pi x\sqrt{1+e^{2x}}\,dx$

(c) $\int_1^{e^2} \pi\sqrt{1+4y^2}\,dy$

(d) $\int_1^{e^2} 2\pi y\sqrt{1+(2\ln y)^2}\,dy$

(e) $\int_0^1 2\pi x\sqrt{1+4e^{2x}}\,dx$

20. The series $\sum_{n=1}^{\infty}(3-\sqrt[n]{3})^{\frac{n}{3}}$ is

(a) divergent by the root test

(b) convergent by the root test

(c) a series for which the root test is inconclusive

(d) a divergent geometric series

(e) convergent by using the comparison test

21. $\displaystyle\int \frac{1}{x\sqrt{x^2+1}}\,dx =$

(a) $\ln|1-\sqrt{x^2+1}| - \ln|x| + C$

(b) $x - \ln|1-\sqrt{x^2+1}| + C$

(c) $\frac{1}{2}x^2 + \ln(1+\sqrt{x^2+1}) + C$

(d) $\ln|\sqrt{1+x^2}+x| + C$

(e) $\dfrac{1-\sqrt{x^2+1}}{x} + C$

22. The interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{n(2x-1)^n}{3^n}$$

is

(a) $(-1, 2)$

(b) $(-1, 2]$

(c) $(-2, 1]$

(d) $[1, 2)$

(e) $(-2, 2)$

23. Which one of the following statements is **FALSE:**
$a_n > 0$ for $n \geq 1$. CC: conditionally convergent

(a) If $\sum_{n=1}^{\infty} a_n$ is divergent, then $\sum_{n=1}^{\infty} \frac{1}{a_n}$ is convergent.

(b) If $\sum_{n=1}^{\infty} a_n$ is convergent, then $\sum_{n=1}^{\infty} \frac{1}{a_n}$ is divergent.

(c) If $\sum_{n=1}^{\infty} (-1)^n a_n$ is CC, then $\sum_{n=1}^{\infty} a_n$ is divergent.

(d) If $\sum_{n=1}^{\infty} a_n$ is convergent, then $\sum_{n=1}^{\infty} (-1)^n a_n$ is convergent.

(e) If $\sum_{n=1}^{\infty} (-1)^n a_n$ is convergent, then $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ is convergent.

24. The improper integral $\int_1^{\infty} x(1+x^2)^{2p+1}\, dx$ is **convergent** if

(a) $p < -1$

(b) $p > -1$

(c) $p < 0$

(d) $p > 0$

(e) $p > -\frac{1}{2}$

25. Using the binomial series, we have, for $|x| < \frac{\sqrt{2}}{2}$,

$$\sqrt{1+2x^2} =$$

(a) $1 + x^2 - \frac{1}{2}x^4 + \frac{1}{2}x^6 - \ldots$

(b) $1 + \frac{1}{2}x^2 + \frac{1}{4}x^4 + \frac{1}{8}x^6 + \ldots$

(c) $1 + 2x^2 - \frac{1}{2}x^4 + \frac{1}{16}x^6 - \ldots$

(d) $1 - x^2 + \frac{1}{2}x^4 - \frac{1}{4}x^6 + \ldots$

(e) $1 + 2x - 4x^2 + 8x^3 - \ldots$

26. The series $\sum_{n=0}^{\infty} \frac{(\sin 3)^{2n}}{\csc^2 3}$ is

(a) convergent and its sum is $\tan^2 3$

(b) convergent and its sum is $(\tan^2 3)(1 - \sin 3)$

(c) convergent and its sum is $\frac{1}{1 - \sin 3}$

(d) convergent and its sum is $\sec^2 3$

(e) divergent

27. $\int_0^1 \cos(x^3)dx =$

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(6n+1)\cdot(2n)!}$

(b) $\sum_{n=0}^{\infty} \frac{-1}{(6n+1)\cdot(2n)!}$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}$

(d) $\sum_{n=0}^{\infty} \frac{n^3}{n!}$

(e) $\sum_{n=0}^{\infty} \frac{(-1)^n \cdot 6^n}{(2n)!}$

28. For some suitable values of x, we have

$$\sum_{n=1}^{\infty} n \cdot 2^n x^n =$$

[Hint: Use the series representation $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, |x| < 1$]

(a) $\frac{2x}{(1-2x)^2}$

(b) $\frac{x}{(1+2x)^2}$

(c) $\frac{x}{1-2x^2}$

(d) $\frac{x^2}{2-x}$

(e) $\frac{x}{(2-x)^2}$

Q	MM	V1	V2	V3	V4
1	a	c	d	a	e
2	a	c	e	b	d
3	a	c	c	e	d
4	a	c	d	c	a
5	a	a	c	d	e
6	a	d	c	b	e
7	a	b	d	a	c
8	a	a	d	c	d
9	a	a	e	a	c
10	a	b	c	a	c
11	a	a	c	d	a
12	a	b	a	e	a
13	a	a	a	d	c
14	a	c	e	a	d
15	a	c	d	e	a
16	a	c	c	e	b
17	a	a	a	e	d
18	a	a	c	e	e
19	a	d	d	a	b
20	a	b	a	d	d
21	a	b	e	c	b
22	a	a	a	b	c
23	a	a	d	e	d
24	a	c	a	c	a
25	a	d	a	e	e
26	a	d	b	e	b
27	a	a	a	c	e
28	a	c	b	a	b