October 15,2012

QUIZ♯2 Math102-sec14.

Net Time Allowed: 20 minutes

Name: **ID ♯:** **section:**

Exercise1: (06 pts)

Find the average value of the function $f(x) = x\sec^2(2x)$ on the interval $[0, \frac{\pi}{8}]$.

solution:

$f_{ave} = (01)\ \frac{1}{\frac{\pi}{8}-0}\int_0^{\frac{\pi}{8}} x\sec^2(2x)\,dx$, We will apply integration by Part:

Let $\left|\begin{matrix} u = x \\ dv = \sec^2(2x)dx \end{matrix}\right. \overset{(01)}{\Rightarrow} \left|\begin{matrix} du = dx \\ v = \frac{1}{2}\tan(2x) \end{matrix}\right.$, Thus:

$f_{ave} = (01)\ \frac{8}{\pi}\left(\left[\frac{x}{2}\tan(2x)\right]_0^{\frac{\pi}{8}} - \frac{1}{2}\int_0^{\frac{\pi}{8}}\tan(2x)\,dx\right)$ (02) $= \frac{4}{\pi}\left(\frac{\pi}{8}\tan\left(\frac{\pi}{4}\right) - \int_0^{\frac{\pi}{8}}\frac{\sin(2x)}{\cos(2x)}\,dx\right)$

Hence $\boxed{f_{ave} = \frac{1}{2} - \frac{\ln 2}{\pi}}$ (01)

Exercise2: (04pts)

Use the method of Shell to find the volume of the solid obtained by rotating the region bounded by $y = x^2 - 1, y = 0, x = \frac{1}{2}, x = 1$ rotated about the line x=1.

solution:

Set $u = x - 1$, So Rotating about $x = 1$ (01) is equivalent to Rotating about $u = 0$ (y-axis). Thus:

$V = \int_{\substack{x=\frac{1}{2}\\ u=-\frac{1}{2}}}^{\substack{x=1\\ u=0}} 2\pi u\left((u+1)^2 - 1\right)du \overset{(01)}{=} 2\pi\int_{-\frac{1}{2}}^{0} u(u^2+2u)\,du$, Hence:

$V = 2\pi\left[\frac{1}{4}u^4 + \frac{2}{3}u^3\right]_{-\frac{1}{2}}^{0} \overset{(01)}{=} 2\pi\left(-\frac{1}{4}\cdot\frac{1}{16} + \frac{2}{3}\cdot\frac{1}{8}\right) = \frac{13}{96}\pi$ (01)