December 12,2012

QUIZ♯4 Math102-sec12.

Net Time Allowed: 20 minutes

Name: ID ♯ : Serial ♯:

Exercise1: (03pts)

Let $\{S_n\}_{n\geq 0}$ be the sequence of partial sums of the series $\sum_{n=1}^{\infty} \frac{1}{n^2+3n+2}$.

a)- Find a formula for $S_n$.

$$\frac{1}{n^2+3n+2} = \frac{1}{(n+1)(n+2)} = \frac{1}{n+1} - \frac{1}{n+2} \quad 01$$

$$S_n = \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots + \left(\frac{1}{n+1} - \frac{1}{n+2}\right)$$

Thus $S_n = \frac{1}{2} - \frac{1}{n+2}$ 01

b)- Use part (a) to find the sum of the series.

$$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(\frac{1}{2} - \frac{1}{n+2}\right) = \frac{1}{2}$$

Therefore. $\boxed{S = \frac{1}{2}}$ 01

Exercise2: (07)

Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}+2^n}{3^n}$ is convergent or divergent. If it is convergent, find its sum.

Determine whether the series is convergent or divergent. If it is convergent, find its sum:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1} + 2^n}{3^n}$ (07pts)

$$\sum_{n\geq 1}^{\infty} \frac{(-1)^{n-1} + 2^n}{3^n} \overset{01}{=} \sum_{n\geq 1}^{\infty} \frac{(-1)^{n-1}}{3^n} + \sum_{n\geq 1}^{\infty} \left(\frac{2}{3}\right)^n.$$

01

⊛ The Series $\displaystyle\sum_{n\geq 1}^{\infty} \frac{(-1)^{n-1}}{3^n}$ is a Geometric Series with $r = -\frac{1}{3}$, Therefore it is Convergent $\left(|r| = \frac{1}{3} < 1\right)$ and has Sum: $\displaystyle\sum_{n\geq 1}^{\infty} \frac{1}{3}\left(-\frac{1}{3}\right)^{n-1} = \frac{1}{3} \cdot \frac{1}{1-\left(-\frac{1}{3}\right)} \overset{01}{=} \frac{1}{4}$ 01

⊛ The Series $\displaystyle\sum_{n\geq 1}^{\infty} \left(\frac{2}{3}\right)^n$ is a Convergent Geometric Series $\left(r = \frac{2}{3}\right)$ with Sum $= 2$ 01

Hence The Given Series is Convergent (01) with Sum: (01) $S = \frac{1}{4} + 2 = \frac{9}{4}$.