October 15,2012

**QUIZ#2 Math102-sec12.**

**Net Time Allowed: 20 minutes**

**Name:** **ID #:** **section:**

**Exercise1:** (06 pts)

Evaluate the integral: $I = \int e^{2\theta} \sin(3\theta)\, d\theta$.

**solution:**

We will apply integration by Parts: Let (01) $u = e^{2\theta} \to du = 2e^{2\theta} d\theta$, $dv = \sin(3\theta) \to v = -\frac{1}{3}\cos(3\theta)$

Thus: $I = -\frac{1}{3} e^{2\theta}\cos(3\theta) + \frac{2}{3}\int e^{2\theta}\cos(3\theta)\, d\theta$ (01); Set again $u = e^{2\theta} \to du = 2e^{2\theta}$, $dv = \cos(3\theta) \to v = \frac{1}{3}\sin(3\theta)$ (01)

Then: $L = \frac{1}{3} e^{2\theta}\sin(3\theta) - \frac{2}{3} I$ (01), Hence

$$I = \frac{1}{3} e^{2\theta}\left(\frac{2}{3}\sin(3\theta) - \cos(3\theta)\right) - \frac{4}{9} I \quad (01)$$

$$\boxed{I = \frac{3}{13} e^{2\theta}\left(\frac{2}{3}\sin(3\theta) - \cos(3\theta)\right) + C} \quad (01)$$

**Exercise2:** (04 pts)

Use the method of Shell to find the volume of the solid obtained by rotating the region bounded by $y = x^2 - 1, y = 0, x = \frac{1}{2}, x = 1$ rotated about the line x=1.

**solution:**

Set $u = x - 1$, So Rotating about $x = 1$ is equivalent to (01) Rotating about $u = 0$ (y-axis). Thus:

$$V = \int_{x=\frac{1}{2}}^{x=1} 2\pi u\left((u+1)^2 - 1\right) du \quad (01) = 2\pi \int_{u=-\frac{1}{2}}^{u=0} u(u^2 + 2u)\, du \quad (01) = 2\pi\left[\frac{1}{4}u^4 + \frac{2}{3}u^3\right]_{-\frac{1}{2}}^{0}$$

Hence $$\boxed{V = 2\pi\left(-\frac{1}{4}\cdot\frac{1}{16} + \frac{2}{3}\cdot\frac{1}{8}\right) = \frac{13}{96}\pi} \quad (01)$$