The German mathematician Karl Weierstrass(1815-1897) noticed that the substitution $t = tan(\frac{x}{2})$ will convert any rational function of $\sin x$ and $\cos x$ into an ordinary rational function of t.

a) If $t = tan(\frac{x}{2})$, $-\pi < x < \pi$, sketch a right triangle or use trigonometric identities to show that

$$\cos(\frac{x}{2}) = \frac{1}{\sqrt{1+t^2}}, \quad \sin(\frac{x}{2}) = \frac{t}{\sqrt{1+t^2}}$$

b) Show that

$$\cos x = \frac{1-t^2}{1+t^2}, \quad \sin x = \frac{2t}{1+t^2}$$

c) Show that

$$dx = \frac{2}{1+t^2}\,dt$$