1. The sum of all values of k for which $y = k$ is a horizontal asymptote to the graph of the function $f(x) = 2 + \left(\dfrac{8x^4 + x^2 - 1}{27x^4 + 8}\right)^{1/3}$ is

   (a) $\dfrac{8}{3}$

   (b) 2

   (c) $\dfrac{62}{27}$

   (d) $\dfrac{5}{2}$

   (e) 0

2. Evaluate $\lim\limits_{x \to 0} \dfrac{\sin 3x - 3x - x^2}{1 - \cos 2x}$.

   (a) $-\dfrac{1}{2}$

   (b) $\dfrac{2}{3}$

   (c) -2

   (d) $-\dfrac{2}{3}$

   (e) Does not exist

3. If the radius of a circle is increased from 3 m to 3.06 m, then the estimated percentage change in the area of the circle is

(a) 4%

(b) 6%

(c) 9%

(d) 2%

(e) 0.36%

4. Using the midpoint rule and 3 rectangles of equal base, the estimated area under the curve $y = \sin(\pi x)$ between $x = 0$ and $x = 1$ is equal to

(a) $\frac{2}{3}$

(b) $\frac{1}{3}(1 + \sqrt{3})$

(c) $\frac{1}{\sqrt{3}}$

(d) $\frac{2}{\pi}$

(e) 1

5. $\int \left(4\sin 2x - \frac{9}{e^{3x}}\right) dx =$

(a) $-2\cos 2x + \frac{3}{e^{3x}} + C$

(b) $-8\cos 2x - \frac{27}{e^{3x}} + C$

(c) $-2\cos 2x + \frac{27}{e^{3x}} + C$

(d) $-8\cos 2x + \frac{3}{e^{3x}} + C$

(e) $2\cos 2x - \frac{3}{e^{3x}} + C$

6. Newton's method is used to estimate the x-coordinate of the point of intersection of the curves $y = x^3 + 3x$ and $y = 4x + 1$. If we start with $x_0 = 1$, then $x_1 =$

(a) 1.5

(b) 1.05

(c) 1.005

(d) 1.4

(e) 1.75

7. $\frac{d}{dx}\left[\csc^2(e^x) - \cot^2(e^x)\right] =$

(a) 0

(b) e^x

(c) $3e^x \cot(e^x) \cdot \csc^2(e^x)$

(d) $-2e^x$

(e) $-e^x \csc(e^x) \cdot \cot(e^x)$

8. If the length x of an edge of a cube decreases at the rate of $0.05\,cm/min$, then the surface area of the cube when $x = 5\,cm$ is decreasing at the rate of

(a) $3\,cm^2/min$

(b) $6\,cm^2/min$

(c) $0.6\,cm^2/min$

(d) $2\,cm^2/min$

(e) $0.2\,cm^2/min$

9. If $y'' = \dfrac{3}{\sqrt{x}}$, $y'(4) = 8$ and $y(1) = 0$, then $y(16) =$

(a) 192

(b) 132

(c) 64

(d) 100

(e) 36

10. The sum of two nonnegative numbers is 27. If p is the product of one of the numbers and the square root of the other, then the largest possible value of p is

(a) 54

(b) 64

(c) 48

(d) 44

(e) 36

11. The function $f(x) = 5x^3 - 3x^5$ has

(a) 2 local extrema and 3 points of inflection

(b) 2 local extrema and 2 points of inflection

(c) 2 critical points and 3 points of inflection

(d) 3 critical points and no points of inflection

(e) 2 local extrema and one point of inflection

12. If $f'(x) = \dfrac{x^2(x-1)}{x+2}$, then which one of the following statement is **TRUE**?

(a) f has one local minimum and no local maxima

(b) f has one local minimum and one local maximum

(c) f is increasing on $(-2, 1)$

(d) f is decreasing on $(1, \infty)$

(e) f has one local maximum and no local minima

13. If c is a number which satisfies the conclusion of the Mean Value Theorem for the function $f(x) = \sin^{-1}\left(\frac{x}{2}\right)$ on the interval $[0, 2]$, then $\pi^2 c^2 + 16 =$

(a) $4\pi^2$

(b) 0

(c) π^2

(d) $2\pi^2$

(e) 4

14. Let f be a differentiable function such that $1 < f'(x) < 3$ for all values of x, then which one of the following statement is TRUE ?

(a) $2 < f(7) - f(5) < 6$

(b) $3 < f(7) - f(5) < 6$

(c) $2 < f(7) - f(5) < 3$

(d) $1 < f(7) - f(5) < 3$

(e) $5 < f(7) - f(5) < 7$

15. The sum of the absolute maximum and the absolute minimum values of the function

$$f(x) = 2\cos x + 2\cos^2 x, \quad \frac{\pi}{2} \leq x \leq 2\pi$$

is

(a) $\frac{7}{2}$

(b) $\frac{5}{2}$

(c) 0

(d) -1

(e) 1

16. Given that $f(x) = \frac{x^2+1}{\sqrt{2x+1}}$, then the number of critical points of f is

(a) 1

(b) 2

(c) 3

(d) 0

(e) 4

17. The linearization of $f(x) = x^x$ at $x = 1$ is given by

(a) $L(x) = x$

(b) $L(x) = x + 1$

(c) $L(x) = 2x - 1$

(d) $L(x) = -x + 2$

(e) $L(x) = x^x$

18. If $f(x) = \log_2(8x^{\ln 2})$ then $f'\left(\frac{1}{2}\right) =$

(a) 2

(b) $\ln 2$

(c) $-\ln 2$

(d) -2

(e) 8

19. Evaluate $\lim_{x\to-\infty} \dfrac{-3x+1}{\sqrt{4x^2+6x+1}}$.

(a) $\dfrac{3}{2}$

(b) $\dfrac{-3}{2}$

(c) ∞

(d) $-\infty$

(e) $\dfrac{-3}{4}$

20. The equation of the tangent line to the curve $y^{\sin x} = \tan x$ at the point $\left(\dfrac{\pi}{4}, 1\right)$, is

(a) $y-1 = 2\sqrt{2}\left(x-\dfrac{\pi}{4}\right)$

(b) $y-1 = \dfrac{1}{\sqrt{2}}\left(x-\dfrac{\pi}{4}\right)$

(c) $y-1 = -\sqrt{2}\left(x-\dfrac{\pi}{4}\right)$

(d) $y-1 = \dfrac{1}{2\sqrt{2}}\left(x-\dfrac{\pi}{4}\right)$

(e) $y-1 = -\dfrac{1}{2\sqrt{2}}\left(x-\dfrac{\pi}{4}\right)$

21. If $f(x) = \dfrac{x + |x|}{|x|}$, then which one of the following statements is **TRUE** ?

(a) $\lim\limits_{x \to 0^-} f'(x) = 0$

(b) $\lim\limits_{x \to \infty} f'(x) = 1$

(c) $\lim\limits_{x \to -\infty} f'(x) = 1$

(d) $\lim\limits_{x \to 0^+} f'(x) = \infty$

(e) $\lim\limits_{x \to 0} f'(x) = \pm 1$

22. Which one of the following functions has a graph with a vertical tangent at $x = 0$?

(a) $f(x) = x - x^{1/3}$

(b) $f(x) = x - x^{-1/3}$

(c) $f(x) = \dfrac{1}{|x|}$

(d) $f(x) = x - x^{4/3}$

(e) $f(x) = \dfrac{x^2}{x^4 - x^2}$

23. If $x^4 + y^4 = 16$, then $y'' =$

(a) $\dfrac{-48x^2}{y^7}$

(b) $\dfrac{3x^3}{y^2}$

(c) $4x^3 + 4$

(d) $12(x^2 + y^2)$

(e) $\dfrac{y^3}{4x^3}$

24. Which one of the following statements is **FALSE** about the graph of the function

$$f(x) = -x - \frac{1}{x-1}?$$

(a) The graph of f has exactly one inflection point

(b) The graph of f is concave downward on $(1, \infty)$

(c) The graph of$f$ is concave upward on $(-\infty, 1)$

(d) The line $y = -x$ is an oblique asymptote of the graph of f

(e) The line $x = 1$ is a vertical asymptote to the graph of f

25. The height of a right circular cone of largest volume that can be inscribed in a sphere of radius 1 is
[Hint: The volume of a cone $= \frac{1}{3} \times$ (area of base) $\times$ height]

(a) $\frac{4}{3}$

(b) $\frac{3}{2}$

(c) $\frac{5}{3}$

(d) $\frac{5}{4}$

(e) $\frac{7}{4}$

26. $\sum_{k=3}^{100}(2-4k+k^2)$ is equivalent to

(a) $\sum_{n=1}^{98}(-2+n^2)$

(b) $\sum_{n=1}^{102}(-2+2n+n^2)$

(c) $\sum_{n=1}^{98}(-4+n^2)$

(d) $\sum_{n=1}^{102}(-2+4n+n^2)$

(e) $\sum_{n=1}^{98}(-6+n^2)$

27. Which one of the following statements is **TRUE** about the continuity of the function?

$$f(x) = \begin{cases} \dfrac{x^3 - 1}{x^2 + 3x - 4}, & x \neq 1, -4 \\ 4, & x = 1 \\ 5, & x = -4 \end{cases}$$

(a) f has a removable discontinuity at $x = 1$

(b) f has a jump discontinuity at $x = 1$

(c) f has an infinite discontinuity at $x = 1$

(d) f is continuous at $x = -4$

(e) f has a removable discontinuity at $x = -4$

28. Evaluate $\lim_{x \to \infty} (1 + 4x)^{3/\ln x}$.

(a) e^3

(b) e^{12}

(c) $e^{3/4}$

(d) 1

(e) Does not exist.