Testing Statistical Hypotheses

One Sample Problem:

$\boldsymbol{\sigma}^2$ known, normal population

Null Hypothesis: H_0	Alternative Hypothesis: H_a	Test Statistic	Rejection Region	p-value
$\mu = \mu_0$	$\mu \neq \mu_0$	$z = \dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$	$\lvert z\rvert > z_{\alpha/2}$	$2P(Z > \lvert z\rvert)$
	$\mu < \mu_0$		$z < -z_\alpha$	$P(Z < z)$
	$\mu > \mu_0$		$z > z_\alpha$	$P(Z > z)$

$\boldsymbol{\sigma}^2$ unknown, large sample

Null Hypothesis: H_0	Alternative Hypothesis: H_a	Test Statistic	Rejection Region	p-value
$\mu = \mu_0$	$\mu \neq \mu_0$	$z = \dfrac{\bar{x} - \mu_0}{s / \sqrt{n}}$	$\lvert z\rvert > z_{\alpha/2}$	$2P(Z > \lvert z\rvert)$
	$\mu < \mu_0$		$z < -z_\alpha$	$P(Z < z)$
	$\mu > \mu_0$		$z > z_\alpha$	$P(Z > z)$

$\boldsymbol{\sigma}^2$ unknown, small sample, normal population

Null Hypothesis: H_0	Alternative Hypothesis: H_a	Test Statistic	Rejection Region	p-value
$\mu = \mu_0$	$\mu \neq \mu_0$	$t = \dfrac{\bar{x} - \mu_0}{s / \sqrt{n}}$	$\lvert t\rvert > t_{\alpha/2,n-1}$	$2P(t_{n-1} > \lvert t\rvert)$
	$\mu < \mu_0$		$t < -t_{\alpha,n-1}$	$P((t_{n-1} < t)$
	$\mu > \mu_0$		$t > t_{\alpha,n-1}$	$P((t_{n-1} > t)$

A population proportion, large sample

Null Hypothesis: H_0	Alternative Hypothesis: H_a	Test Statistic	Rejection Region	p-value
$p = p_0$	$p \neq p_0$	$z = \dfrac{p - p_0}{\sqrt{p_0(1 - p_0)/n}}$	$\lvert z\rvert > z_{\alpha/2}$	$2P(Z > \lvert z\rvert)$
	$p < p_0$		$z < -z_\alpha$	$P(Z < z)$
	$p > p_0$		$z > z_\alpha$	$P(Z > z)$

Two Sample Problem:

σ_1^2 and σ_2^2 known, two independent samples, normal populations

Null Hypothesis: H_0	Alternative Hypothesis: H_a	Test Statistic	Rejection Region	p-value
$\mu_1 - \mu_2 = d_0$	$\mu_1 - \mu_2 \neq d_0$	$z = \dfrac{\overline{x_1} - \overline{x_2} - d_0}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$	$\lvert z\rvert > z_{\alpha/2}$	$2P(Z > \lvert z\rvert)$
	$\mu_1 - \mu_2 < d_0$		$z < -z_\alpha$	$P(Z < z)$
	$\mu_1 - \mu_2 > d_0$		$z > z_\alpha$	$P(Z > z)$

σ_1^2 and σ_2^2 unknown, two independent large samples

Null Hypothesis: H_0	Alternative Hypothesis: H_a	Test Statistic	Rejection Region	p-value
$\mu_1 - \mu_2 = d_0$	$\mu_1 - \mu_2 \neq d_0$	$z = \dfrac{\overline{x_1} - \overline{x_2} - d_0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$	$\lvert z\rvert > z_{\alpha/2}$	$2P(Z > \lvert z\rvert)$
	$\mu_1 - \mu_2 < d_0$		$z < -z_\alpha$	$P(Z < z)$
	$\mu_1 - \mu_2 > d_0$		$z > z_\alpha$	$P(Z > z)$

σ_1^2 and σ_2^2 unknown, $\sigma_1^2 = \sigma_2^2$ small samples, two independent samples normal populations

Null Hypothesis: H_0	Alternative Hypothesis: H_a	Test Statistic	Rejection Region	p-value
$\mu_1 - \mu_2 = d_0$	$\mu_1 - \mu_2 \neq d_0$	$t = \dfrac{\overline{x_1} - \overline{x_2} - d_0}{s_p\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$	$\lvert t\rvert > t_{\alpha/2,f}$	$2P(t_f > \lvert t\rvert)$
	$\mu_1 - \mu_2 < d_0$		$t < -t_{\alpha,f}$	$P((t_f < t)$
	$\mu_1 - \mu_2 > d_0$		$t > t_{\alpha,f}$	$P((t_f > t)$

where $$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

and f, the number of degrees of freedom, $= n_1 + n_2 - 2$.