Confidence Interval Estimation

1. Large Sample $100(1-\alpha)\%$ Confidence Interval for the mean μ

   σ known: $$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

   Required sample size to estimate the mean, μ, with a maximum error E and with confidence $1 - \alpha$ $$n = \left(\frac{z_{\alpha/2}\,\sigma}{E}\right)^2$$

   σ unknown: $$\bar{x} \pm z_{\alpha/2}\frac{s}{\sqrt{n}}$$

2. Small Sample $100(1-\alpha)\%$ Confidence Interval for the mean μ of a Normal Population

   σ known: $\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

   σ unknown $\bar{x} \pm t_{\alpha/2,f}\frac{s}{\sqrt{n}}$, the number of degrees of freedom $f = n-1$

3. Large Sample $100(1-\alpha)\%$ Confidence Interval for the difference in the means $\mu_1 - \mu_2$ using two independent samples.

   If σ_1 and σ_2 are known: $$\left(\bar{x}_1 - \bar{x}_2\right) \pm z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

   If σ_1 and σ_2 are unknown: $$\left(\bar{x}_1 - \bar{x}_2\right) \pm z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

4. Small Samples 100(1 – α)% Confidence Interval for the difference in the means $\mu_1 - \mu_2$ of Two Normal Populations with known variances, σ_1^2 and σ_2^2 , using two independent samples.

$$\left(\overline{x_1} - \overline{x_2}\right) \pm z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

5. Small Samples 100(1 – α)% Confidence Interval for the difference in the means $\mu_1 - \mu_2$ of Two Normal Populations with unknown equal variances, $\sigma_1^2 = \sigma_2^2$, using two independent samples.

$$\left(\overline{x_1} - \overline{x_2}\right) \pm t_{\alpha/2,f}\ s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}},$$

where $s_p^2 = \dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$

and the number of degrees of freedom $f = n_1 + n_2 - 2$.

6. Large Sample 100(1 – α)% Confidence Interval for *p*, a population proportion

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Required Sample Size to estimate a population proportion, *p*, with a maximum error E and with confidence 1 - α:

- if we have a preliminary estimate $\hat{p}$

$$n = \frac{z_{\alpha/2}^2\,\hat{p}(1-\hat{p})}{E^2}$$

- if we do not have a preliminary estimate $\hat{p}$

$$n_{max} = \frac{z_{\alpha/2}^2}{4E^2}$$