KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICAL SCIENCES
DHAHRAN, SAUDI ARABIA

STAT 319: PROBABILITY & STATISTICS FOR ENGINEERS & SCIENTISTS

Final Exam, Term 112
Time: 7.00- 9.30 p.m., Monday May 21, 2012

Please check/circle the name of your instructor; Write clearly your name, ID, and section number.

Instructors:

□ Anabosi □ Al-Sabah. □ Joarder
□ Muttlak □ Riaz

Student Surname: Anabosi ID# Section #

You are allowed to use electronic calculators and other reasonable writing accessories that help write the exam. Try to define events, formulate problem and solve. See example below.

Example Q:
(3pts) Find the **Area** of a rectangle with perimeter of 30 units and length of 8 units.

Example **Answer with grading point** scheme.

Perimeter $= 2(l + w) = 30 \rightarrow l + w = 15$ (1 pt)
Length $= l = 8 \rightarrow w = 15 - l = 7$ (1pt)
$\Rightarrow$ Area $= l * w = 8*7 = 56$ unit^2. (1 pt)

Do not keep your **mobile** with you during the exam, turn off your mobile and leave it aside.

	Question No	Full Marks	Marks Obtained
	1	10	
10	2	16	
10	3	7	
10	4	10	
17	5	12	
8	6	11	
26	7	19	
	Total	85	

Note: You may assume $\alpha = 0.05$ for testing if not otherwise stated.

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Q1. (2+3+2+3=10)

The thickness (X), in microns, of a sample of metal wires produced in a chip manufacturing process are given in the flowing table:

$\sum x_i f_i$	14	37.5	96	127.5	275
x_i Thickness	7	7.5	8	8.5	Total
f_i Frequency	2	5	12	15	34
$\sum x_i^2 f_i$	98	281.25	768	1083.75	2231

a. Name the table. What is the sample size?

(1) ← A frequency table. $n = 34$ → (1)

b. Determine standard deviation of the thickness of the metal wires.

$$S = \sqrt{\frac{\sum x_i^2 f_i - \frac{(\sum x_i f_i)^2}{n}}{n-1}} = \sqrt{\frac{2231 - \frac{(275)^2}{34}}{33}} \rightarrow (2)$$

$$= \sqrt{\frac{6.7353}{33}} = \sqrt{0.2041} = \boxed{0.4518} \text{ micron} \rightarrow (1)$$

c. Calculate coefficient of Variation and explain it.

$$\bar{x} = \frac{\sum x_i f_i}{n} = \frac{275}{34} = \boxed{8.0882} \text{ microns} \rightarrow (1)$$

$$C.V. = \frac{S}{\bar{x}} = \frac{0.4518}{8.0882} = \boxed{0.0559} \approx 5.59\,\% \; (1)$$

d. What is percentage of metal wires with a thickness of at most 8 microns?

$$\text{Percentage} = \frac{19}{34} = \boxed{55.8824\,\%} \rightarrow (1)$$

(1) (1)

Q2. (4x4=16). Answer the following problems:

a. Extensive experience with fans of a certain type used in diesel engines has suggested that the exponential distribution provides a good model for time until failure. Suppose the mean time until failure is 3 years. What is the probability that the lifetime of a fan exceeds 3 years?

X: Fan lifetime until failure. $X: EXP(\lambda) \rightarrow 1$

$\lambda = \frac{1}{\mu} = \frac{1}{3} = 0.3333$ failure / year. $\rightarrow 1$

$f(x) = \lambda e^{-\lambda x} = \frac{1}{3} e^{-x/3}, \; x > 0 \rightarrow 1$

$P(X>3) = e^{-3/3} = e^{-1} = \boxed{0.3679} \rightarrow 1$

b. An assembly line is out of control with 20% of the items produced being independently defective, what is the probability that two successive items are defective?

$\underbrace{p = 0.2}_{1} \Rightarrow \underbrace{P(DD) = (0.2)^2}_{2} = \underbrace{0.04}_{1}$

c. A purchaser of electrical components buys them in lots of size 10. It is his policy to inspect 3 components randomly from a lot to accept the lot only if all 3 are non-defective. If there are two defective items in each lot, what proportion of lots does the purchaser reject?

Y: # of defective components out of 3 $\Rightarrow Y: HG(N, n, a) \rightarrow 1$

$N = 10, \; n = 3, \; a = 2$

$P(Accept) = P(Y=0) = \frac{\binom{2}{0}\binom{8}{3}}{\binom{10}{3}} = \frac{56}{120} = 0.4667 \rightarrow 2$

$P(Reject) = 1 - P(Accept) = \boxed{0.5333} \rightarrow 1$

d. The thickness of certain electronic components is normally distributed with the mean of 10 mm and standard deviation 1mm. Find the median for the thickness of these electronic components.

X: Thickness $\Rightarrow X: N(10, 1)$ 4

Median = Mean = Mode = 10 mm.

or

$Pr(X < K) = 0.5$ — 1

$\Rightarrow Pr(Z < \frac{K-10}{1}) = 0.5$ — 1

Table $\Rightarrow Pr(Z < 0) = 0.5$ — 1

So $\frac{K-10}{1} = 0 \Rightarrow K = 10$ which is median. — 1

Q3. (7) + 1 + 2 = 10)

The following sample data were obtained for the curing time, in hours, of an experimental adhesive,

31.5	28.7	42.5	39.3	29.8	31.0	36.3	33.4	35.5	37.2

a. Construct a 98% confidence interval for the mean curing time of the adhesive.

$n = 10$, $\bar{x} = \boxed{34.52}$ hr, $S = \boxed{4.4317}$ hr → 2

Assumptions: ① small sample ② Assume normal population.
③ σ is unknown ⇒ use t-distribution

$1-\alpha = 0.98 \Rightarrow \frac{\alpha}{2} = \frac{0.02}{2} = 0.01 \Rightarrow \boxed{t_{0.01,9} = 2.821}$ → 1

A 98% C.I. for μ is $\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$

2 $= \left[34.52 \pm (2.821)\frac{4.4317}{\sqrt{10}}\right] = [34.52 \pm 3.9534] = [30.5667, 38.4734]$
→ 1 → 1

b. Interpret the interval obtained in (a).

We are 98% Confident that the true (population) mean Curing time of the adhesive is between 30.57 hrs and 38.47 hrs. → 1

c. Use this confidence interval to test the claim that the mean curing time is not equal to 35, at 2% significance level.

① $H_0: \mu = 35$ vs. $H_1: \mu \neq 35$ → 1

② Assumptions: checked in ⓐ.

③ DR: If $\mu_0 \in$ C.I. ⇒ can NOT reject H_0. → 1

④ Decision: Since $\mu_0 = 35 \in$ C.I. ⇒ We can NOT reject H_0.

⑤ Conclusion: The data provide sufficient evidence to Conclude that the mean curing time is 35 hrs.

~~7~~ 7+3 ~~5~~ 5
Q4. (~~7~~+~~5~~=10)

Records indicate that of all vehicles undergoing emissions testing during the previous year, 70% passed on the first try. A random sample of 200 cars tested in a particular county during the current year yields 160 that passed on the first try.

a. Does this suggest that the true proportion for this county during the current year is larger than the previous proportion? Report your p-value.

$P = 0.7$, $n = 200$, $x = 160 \Rightarrow \hat{p} = \frac{x}{n} = \frac{160}{200} = 0.8$ (1)

Assumptions: since $np = 200(0.8) = 160 \geq 5$ and $nq = 200(0.2) = 40 \geq 5$ then we can use the normal approximation to the binomial.

$\Rightarrow \hat{p} \sim N(P, \frac{PQ}{n})$ $\Rightarrow$ ① $H_0: P = 0.7$ vs. $H_1: P > 0.7$

③ TS: $Z_0 = \frac{\hat{p} - P_0}{\sqrt{\frac{P_0 Q_0}{n}}} = \frac{0.8 - 0.7}{\sqrt{\frac{(0.7)(0.3)}{200}}} = \frac{0.1}{0.0324} = 3.086 \rightarrow$ (2)

(1)

④ p-value $= P(Z > z_0) = P(Z > 3.086) = P(Z < -3.086) = 0.001$ (2)

⑤ Since p-value $= 0.001 < 0.05 = \alpha \Rightarrow$ Reject H_0 $\rightarrow$ (1)

b. Determine the sample size to estimate proportion of all vehicles to within ±3% with 95% confidence level.

$$n = \left(\frac{Z_{\alpha/2}\sqrt{\hat{p}\hat{q}}}{e}\right)^2 \quad — (1)$$

$$= \left(\frac{1.96}{0.03}\sqrt{(0.8)(0.2)}\right)^2 \quad — (1)$$

$$= 682.95 \simeq 683 \quad — (1)$$

Another approach: i.e using $P = 0.5$ for an upper bound

$$n \leq \left(\frac{Z_{\alpha/2}}{e}\sqrt{(0.5)(0.5)}\right)^2 \quad — (2)$$

$$\simeq 1068 \rightarrow (1)$$

57
8 2
Q5. (~~9~~+~~7~~=1~~0~~)

Fifteen specimens of a new computer chip were tested for speed in a certain application, along with 15 specimens of chips with the old design. The average speed, in MHz, for the new chips was 495.6, and the standard deviation was 19.4. The average speed for the old chips was 481.2, and the standard deviation was 14.3.

a. Can you conclude that the mean speed for the new chips is greater than that of the old chips? State the appropriate null and alternate hypotheses, test statistics decision and final recommendation.

1

① $H_0: \mu_N = \mu_O$ vs. $H_1: \mu_N > \mu_O$

② Assumptions. a. Indep. samples.
b. Small " "
c. Assume normal populations.
d. Unknown σ's.
e. Equal σ's.

③ Test statistic:

$$t_o = \frac{(\bar{x}_N - \bar{x}_O) - \delta_o}{s_p \sqrt{\frac{1}{n_N} + \frac{1}{n_O}}}$$

$$= \frac{(495.6 - 481.2) - 0}{(17.0419)\sqrt{\frac{1}{15} + \frac{1}{15}}}$$

$$= 2.341$$

2

④ Critical Value: t_{α, n_1+n_2-2}
$\Rightarrow t_{0.05, 28} = 1.701$ 1

1 ⑤ Decision Rule: If $t_o > t_\alpha$ we rej. H_0

	New	Old
n	15	15
$\bar{x}$	495.6	481.2
s	19.4	14.3

$$s_p = \sqrt{\frac{(n_N - 1)s_N^2 + (n_O - 1)s_O^2}{n_N + n_O - 2}}$$

$$= \sqrt{\frac{14(19.4)^2 + 14(14.3)^2}{28}}$$

$$= \sqrt{290.425} = 17.0419$$ 1

⑥ Decision: 1
since $t_o = 2.341 > 1.701 = t_\alpha$
$\Rightarrow$ Reject H_0.

⑦ Conclusion: Yes, we can 1
Conclude that New chips are faster than the Old ones.

b. What assumptions you have made for part(a)?

1 First: We have to assume samples are Independent.

1 Second: " " " " populations are Normal.

[redacted] Third: " " " " σ's are Equal.

15

Q6. (7+3=10)

A random sample of 100 automobile owners shows that, an automobile is driven on the average 23,500 kilometers per year with a standard deviation of 3900 kilometers. Assume the distribution of measurements to be normal.

a. Construct a 99% confidence interval for the average number of kilometers an automobile is driven. Also interpret this confidence interval in the context of the given problem

$n = 100, \quad \bar{x} = 23500 \quad / S = 3900$

A $(1-\alpha)100\%$ C-I- for μ is $\bar{X} \pm Z_{\alpha/2} \frac{S}{\sqrt{n}}$ (1)

$1-\alpha = 0.99 \Rightarrow \frac{\alpha}{2} = \frac{0.01}{2} = 0.005 \Rightarrow Z_{0.005} = 2.575$ (1)

$\Rightarrow$ A 99% C-I- for μ is $\left[23500 \pm (2.575)\frac{3900}{\sqrt{100}}\right]$ (2)

(1) $= [23500 \pm 1004.25] = [22495.75, \ 24504.25]$ (1)

That is we are 99% confident that the true average (1) number of kilometers driven is between 22495.75 km and 24504.25 km.

b. What can we assert with 99% confidence about the possible size of our error if we estimate the average number of kilometers driven to be 23,500 kilometers per year?

$\bar{x} = 23500, \quad Z_{0.005} = 2.575, \quad S = 3900, \quad n = 100$

$\Rightarrow E \leq \frac{Z_{\alpha/2} \cdot S}{\sqrt{n}} = \frac{(2.575)(3900)}{10} = 1004.25$ km

(1) (1) (1)

Q.7 (3+2+3+5+6 =19) The tensile strength (Y) of a paper product is related to the amount of hardwood in the pulp (X). Ten samples are produced in the pilot plant, and the data obtained are shown in the following table.

x	10	15	15	20	20	20	25	25	28	30
y	160	171	175	182	184	181	186	193	195	200

where $\sum x_i = 208$, $\sum y_i = 1827$, $\sum x_i^2 = 4684$, $\sum y_i^2 = 335077$, $\sum x_i y_i = 38665$, $SSR = 1230.607$

$SSE = 53.493$

$S_e = 2.586$

a. Calculate the least squares estimates for the regression equation tensile relating strength to percent hardwood.

$\hat{Y} = a + bx$

$b = \frac{S_{xy}}{S_{xx}} = \frac{663.4}{357.6} = \boxed{1.86}$

1.855 → 1

$a = \bar{y} - b\bar{x} = 182.7 - (1.86)(20.8) = \boxed{144.01}$ → 1

144.113

$S_{xy} = 38665 - \frac{(208)(1827)}{10} = 663.4$

$S_{xx} = \Sigma x^2 - n\bar{x}^2 = 357.6$

$S_{yy} = \boxed{1284.1 = SST}$

144.43

$\boxed{\hat{Y} = 144.01 + 1.86x}$ → 1

b. Find the error in estimating the tensile strength if the percent hardwood is 20.

$\hat{Y}_{20} = 144.01 + 1.86(20) = 181.12$ → 1

181.216

Error $= \hat{Y}_{20} - Y_{20} = 182 - 181.12 = 0.885$ 0.784 → 1

or $184 - 181.12 = 2.86$ 2.784

-0.12 -0.216

c. Calculate the coefficient of determination for regressing tensile strength to percent hardwood, and explain it.

1 ← $R^2 = \frac{b S_{xy}}{S_{yy}} = \frac{1.86(663.4)}{1284.1} = \boxed{0.958}$ → 1

95.8% of the variation in tensile strength is explained by the amount of hardwood. 1

d. Using a 99% confidence level, estimate a future tensile strength if the percent hardwood is 16.

A $100(1-\alpha)\%$ P.I for Y_{x_0} is $\boxed{\hat{y}_0 \pm t_{\alpha/2}\sqrt{\left(1+\frac{1}{n}+\frac{(x_0-\bar{x})^2}{S_{xx}}\right)s_e^2}}$ (1)

Here $\alpha/2 = 0.005 \Rightarrow t_{\alpha/2} = 3.355$ as $v = n-2 = 8$

(1) $s_e^2 = \frac{SSE}{n-2} = \frac{50.176}{8} = \boxed{6.272}$ 6.687

$SSE = S_{yy} - bS_{xy}$
$= 1284.1 - 1.855(663.4)$
$= 50.176 = \boxed{53.493}$

$s_e = 2.586$

$\hat{Y}_{16} = 173.69 \rightarrow$ (1)

173.795

So we have

$173.69 \pm 3.355\sqrt{\left(1+\frac{1}{10}+\frac{(16-20.8)^2}{357.6}\right)6.272}$ (1)

2.790

173.795 $\pm$ 9.36

173.69 ± 9.07

$[164.62, 182.75]$

164.435, 183.155

$s_e\sqrt{\frac{1}{n}+\frac{(x_0-\bar{x})^2}{S_{xx}}} = 1.049$

173.795 ± 3.518

$[170.277, 177.313]$ (1)

e. If the engineer thinks: the higher the percent hardwood, the more the tensile strength. Do the data support his thought at 5% level of significance?

(2)

→ $H_0: \beta = 0$, $H_1: \beta > 0$ (1)

→ Test statistic: $t = \frac{b-\beta_0}{\sqrt{s_e^2/S_{xx}}} = \frac{1.86 - 0}{\sqrt{6.272/357.6}} = 14.01$

1.855 ; 6.687 ; 13.565

→ Critical Region: $\alpha = 0.05$, $v = n-2 = 8$, $t_{\alpha,8} = 1.86$

so Rejection region is $t \geq 1.86$ (1)

→ Decision: As $14.01 > 1.86$ so Reject H_0 (1)

13.565

→ Conclusion: The data support the thought of engineer. (1)