KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICAL SCIENCES
DHAHRAN, SAUDI ARABIA

STAT 319: PROBABILITY & STATISTICS FOR ENGINEERS & SCIENTISTS

Second Major, Term 112
Time: 6:30 p.m. to 8:00 pm, April 14, 2012

Please Check/circle the name of your instructor; Write clearly your name, ID, and section number.

Instructors:

□ Anabosi □ Al-Sabah. □ Joarder
□ Muttlak □ Riaz

Student Surname: KEY ID# Section #

You are allowed to use electronic calculators and other reasonable writing accessories that help write the exam. Try to define events, formulate problem and solve. See example below.

Example Q:
(3pts) Find the **Area** of a rectangle with perimeter of 30 units and length of 8 units.

Example Answer with grading point scheme.

Perimeter $=2(l + w) = 30 \rightarrow l + w = 15$ (1 pt)
Length $= l = 8 \rightarrow w = 15-l = 7$ (1pt)
$\Rightarrow$ Area $= l*w = 8*7 = 56$ unit^2. (1 pt)

Do not keep your **mobile** with you during the exam, turn off your mobile and leave it aside.

Question No	Full Marks	Marks Obtained
1	10	
2	7	
3	5	
4	10	
5	13	
Total	45	

50 minutes

20

1. [2+3+3+2 =10] Samples of 20 parts from a metal punching process are selected every hour. Typically, 1% of the parts require rework. Let X denote the number of parts in the sample of 20 that require rework.

(a) What is the probability that X exceeds 1?

(b) Find the mean and standard deviation?

(c) A process problem is suspected if X exceeds its mean by more than three standard deviations, what is the probability that X exceeds its mean by more than three standard deviations?

(d) If the rework percentage increases to 4%, what is the probability that X less than 3?

Solution:

$P(R) = p = 0.01$, X: # of parts that need rework

$n = 20$, $X: B(20, 0.01) \Rightarrow f(x) = \binom{20}{x}(0.01)^x(0.99)^{20-x}$

(a) $P(X>1) = 1 - P(X \le 1) = 1 - [f(0) + f(1)]$

$\boxed{1}$ $= 1 - [0.99^{20} + 20(0.01)(0.99)^{19}]$

$\boxed{1}$ $= 1 - 0.9831 = \boxed{0.0169}$

$\boxed{1}$ (b) $\mu_X = np = 20(0.01) = \boxed{0.2}$ parts

$\boxed{2}$ $\sigma_X = \sqrt{npq} = \sqrt{20(0.01)(0.99)} = \boxed{0.445}$ parts

$\boxed{1}$ (c) $P(X > \mu + 3\sigma) = P(X > 0.2 + 3(0.445))$

$\boxed{1}$ $= P(X > 1.535) = P(X > 1)$

$\boxed{1}$ $= \boxed{0.0169}$

(d) If $p = 0.04$, $n = 20 \Rightarrow f(x) = \binom{20}{x}(0.04)^x(0.96)^{20-x}$

$P(X<3) = f(0) + f(1) + f(2)$

$\boxed{1}$ $= \binom{20}{0}(0.04)^0(0.96)^{20} + \binom{20}{1}(0.04)^1(0.96)^{19} + \binom{20}{2}(0.04)^2(0.96)^{18}$

$= 0.96^{18}[(0.96)^2 + 20(0.04)(0.96) + 190(0.04)^2]$

$= 0.4420 + 0.3683 + 0.1458$

$\boxed{1}$ $= \boxed{0.9561}$

3

2. [3+4=7] In the inspection of tin plates produced by a continuous electrolytic process, the probability of spotting x imperfections per minute is given by the following discrete probability function:

$$f(x) = \frac{1}{x!}e^{-1}, \quad x = 0, 1, \cdots, \infty$$

a. What is the probability of having no imperfections per minute?
b. What is the probability of having no imperfections per 5 minutes?

Solution:

Let X: # of imperfections per t minutes.

$\Rightarrow X : Po(1)$; $\lambda = 1$ $\Rightarrow f(x) = \dfrac{e^{-t}\, t^{x}}{x!}$

$\boxed{3}$ (a) $P(X = 0 \mid t = 1) = e^{-1} = \boxed{0.3679}$

(b) $P(X = 0 \mid t = 5) = e^{-5} = \boxed{0.0067}$

$\boxed{2}$ $\boxed{2}$

5

3. [3+2=5] In a large lot of polished steel shafts, 5% have surfaces that are rough. What is the probability that the first shaft with rough surface is the 10^{th} one selected? What assumptions are you making about the size of the lot, and the selection process?

Solution:

Let X : # of shafts inspected untill the 1st rough shaft.

If $p = 0.05 \Rightarrow X : G(0.05) \Rightarrow f(x) = (0.05)(0.95)^{x-1}$

$\boxed{2}$ $P(X=10) = (0.05)(0.95)^9 = 0.0315 \rightarrow \boxed{1}$

$\boxed{1}$ Assumptions: ① Lot size is assumed infinite.

② Selection is done without replacement.

$\boxed{1}$ Items are selected independently

10

4. [3+3+4=10]The length of time it takes students to complete an exam is given by a random variable, Y (measured in hours), which has a probability density function given by:

$$f(y)=\begin{cases} ay, & 1\le y\le 5 \\ 0, & elsewhere \end{cases}$$

a. Find the value of a.
b. Find the mean number of hours that the students will take to complete this exam.
c. Thirty six students took the exam. What is the probability the sample mean of the time to complete the exam is more than 3.5 hours?

Solution:

1 (a) If $f(y)$ is a pdf then $\int_1^5 f(y)\,dy = 1$

1 $\Rightarrow a\int_1^5 y\,dy = 1 \Rightarrow \frac{a}{2} y^2\Big]_1^5 = 1 \Rightarrow \frac{a}{2}(25-1) = 1$

1 $\Rightarrow 12a = 1 \Rightarrow \boxed{a = \frac{1}{12}}$

1 (b) $E(Y) = \mu_Y = \int_1^5 y\,f(y)\,dy = \frac{1}{12}\int_1^5 y^2\,dy$

1 $= \frac{1}{36}\, y^3\Big]_1^5 = \frac{1}{36}[125-1] = \frac{124}{36} = \boxed{3.4444}$ hours.

1

1 (c) $P(\bar{Y} > 3.5) = P\left(\frac{\bar{Y}-\mu_Y}{\sigma_Y/\sqrt{n}} > \frac{3.5-3.4444}{\sigma_Y/\sqrt{36}}\right)$

$E(Y^2) = \frac{1}{12}\int_1^5 y^3\,dy = \frac{1}{48}\, y^4\Big]_1^5 = \frac{625}{48} = 13.0208$

$\sigma_Y^2 = E(Y^2) - (EY)^2 = \frac{625}{48} - \left(\frac{124}{36}\right)^2 = 1.1566$

1 $\Rightarrow \sigma_Y = \sqrt{1.1566} = 1.0755$

$\Rightarrow P(\bar{Y} > 3.5) = P\left(Z > \frac{6(3.5-3.4444)}{1.0755}\right)$ by CLT 1

$= P(Z > 0.31) = P(Z < -0.31)$

1 $= \boxed{0.3782}$

12

5. **[3+3+4+3=13]** The width (in inches) of a slot of a duralumin forging is normally distributed with mean 0.9 inch and some standard deviation inch. The specification limits were given as 0.9000 ± .0050.

a. What percentage of forgings will be defective (out of specification) if the standard deviation width of a slot of a duralumin forging is 0.003 inch?

b. If 95% widths are less than k inches with a standard deviation of 0.001 inch, determine k.

c. Find the value of *standard deviation* for which 99% of the forgings are within the specifications, when the widths are normally distributed with *mean* 0.9 inch.

d. A random sample of 9 forgings is randomly selected from normally distributed process with mean 0.9 inch and standard deviation 0.002 inch, what is probability that the sample mean width exceeds 0.9 inch?

Solution:

Let X: width of the Slot $\Rightarrow X : N(0.9, \sigma^2)$

(a) $\sigma = 0.003 \Rightarrow P(X > 0.905 \text{ or } X < 0.895)$

$\boxed{1} \rightarrow = 2P(X > 0.905) = 2P\left(Z > \frac{0.905 - 0.9}{0.003}\right)$

$\boxed{1} \rightarrow = 2P(Z > 1.67) = 2P(Z < -1.67)$

$\boxed{1} \rightarrow = 2(0.0475) = \boxed{0.095} \Rightarrow 9.5\%$

$\boxed{1}$ (b) $\rightarrow P(X < k) = 0.95 \Rightarrow$

$\boxed{1} \rightarrow P\left(Z < \frac{k - 0.9}{0.001}\right) = 0.95$

$\boxed{1} \rightarrow \Rightarrow \frac{k - 0.9}{0.001} = 1.645 \Rightarrow k = 1.645(0.001) + 0.9$

$\Rightarrow k = \boxed{0.901645}$ inch.

$\boxed{1}$ (c) $\rightarrow P(0.895 < X < 0.905) = 0.99 \Rightarrow P(X < 0.905) = 0.995$

$\boxed{1} \rightarrow \Rightarrow P\left(Z < \frac{0.905 - 0.9}{\sigma}\right) = 0.995$

$\boxed{1} \rightarrow \Rightarrow \frac{0.905 - 0.9}{\sigma} = 2.575 \Rightarrow \sigma = \frac{0.905 - 0.9}{2.575} = \boxed{0.001942}$ $\boxed{1}$

(d) $P(\bar{X} > 0.9) = \frac{1}{2}$ by Symmetry.

$\boxed{1} \rightarrow P(\bar{X} > 0.9) = P\left(Z > \frac{0.9 - 0.9}{\frac{0.002}{3}}\right) = P(Z > 0) = \frac{1}{2}$

$\boxed{1}$ $\boxed{1}$