Solution Key.

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICAL SCIENCES
DHAHRAN, SAUDI ARABIA

STAT 319: PROBABILITY & STATISTICS FOR ENGINEERS & SCIENTISTS

First Mid Term Exam, Term 112

Time: 6:00 p.m. to 7:30 p.m., Mar 11 , 2012

Please Check/circle the name of your instructor; Write clearly your name, ID, and section number.

Instructors:

□ Anabosi □ Al-Sabah. □ Joarder
□ Muttlak □ Riaz

Student Surname: ID# Section #

You are allowed to use electronic calculators and other reasonable writing accessories that help write the exam. Try to define events, formulate the problem and solve it. See the example below.

Example Q:
(3pts) Find the **Area** of a rectangle with perimeter of 30 units and length of 8 units.

Example Answer with grading point scheme.

Perimeter $=2(l + w) = 30 \rightarrow l + w = 15$	(1 pt)
Length $= l = 8 \rightarrow w = 15-l = 7$	(1pt)
$\Rightarrow$ Area $= l*w = 8*7 = 56$ unit^2.	(1 pt)

Do not keep your **mobile** with you during the exam, turn off your mobile and leave it aside.

Question No	Full Marks	Marks Obtained
1	19	
2	4	
3	6	
4	6	
5	5	
Total	40	

Q1. (9+3+3+4)=19.

A chemical engineer studied the effect of reflux rate on the yields from a distillation column. Here are the yields

X	0.90	0.90	0.92	0.92	0.92	0.93	0.94	0.94	0.94	0.94	0.95	0.95
	0.95	0.95	0.95	0.95	0.95	0.96	0.96	0.97	0.97	0.97	0.98	0.98

Where $\sum X = 22.69$ $\sum X^2 = 21.4627$

a. Find mean, standard deviation, median and mode of the yields data. Also check the empirical rule for the given dataset.

b. Prepare a frequency distribution of the reflux rates. Use intervals of length 2 (for example first interval is 0.90-0.92).

c. Draw the relative frequency curve of the reflux rates. Comment on the shape of the curve.

d. What proportion of the rates are less than or equal to 0.93? Name this proportion in terms of percentile. **Calculate** the 25th percentile by the formula. Are they the same? Explain if they are different.

Solution: a. $\bar{x} = 0.94541666\ldots$ (1)

$s = \sqrt{\frac{1}{n-1}\sum(x-\bar{x})^2} \approx 0.022062986$ (2)

Since $0.50(1+n) = 0.50(1+24) = 12.5$, the median $\tilde{x}$ is $(1-0.50)(.95) + 0.50(.95) = 0.95$. (2)

The mode is 0.95. ~~There is a slight skewness to the left.~~ (1)

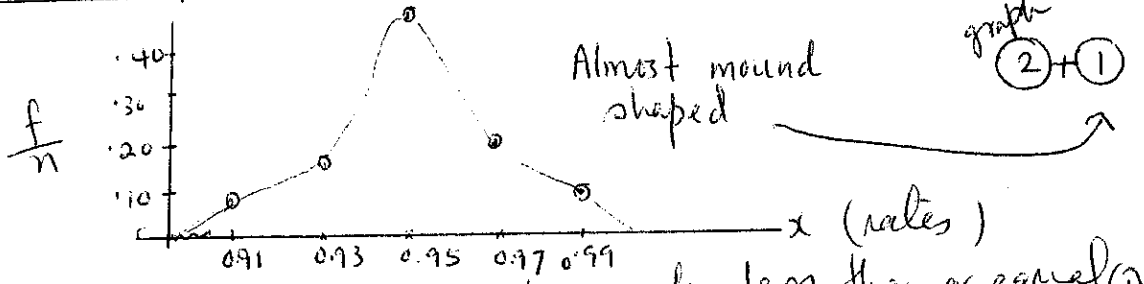
The interval $[\bar{x}-s, \bar{x}+s]$ $[0.923, 0.967]$ contains 14 of the 24 values, making 58%, while the first condition of ER requires 68%. So the ER is not satisfied. (3)

b.

x	[.90, .92)	[.92, .94)	[.94, .96)	[.96, .98)	[.98, 1.00)
f	2	4	11	5	2
f/n	0.08	0.17	0.46	0.21	0.08

(3)

c.



graph (2)+(1)

d. There are 6 out of 24 reflux rates less than or equal to 0.93. This makes 25% of the smallest rates. (i.e. 25th percentile) (1)

Since $0.25(1+24) = 6.25$, the 25th percentile is given by $0.75(0.93) + .25(0.94) \approx 0.9325$. (2)

The difference is due to the fact that the formula is approximate. (1)

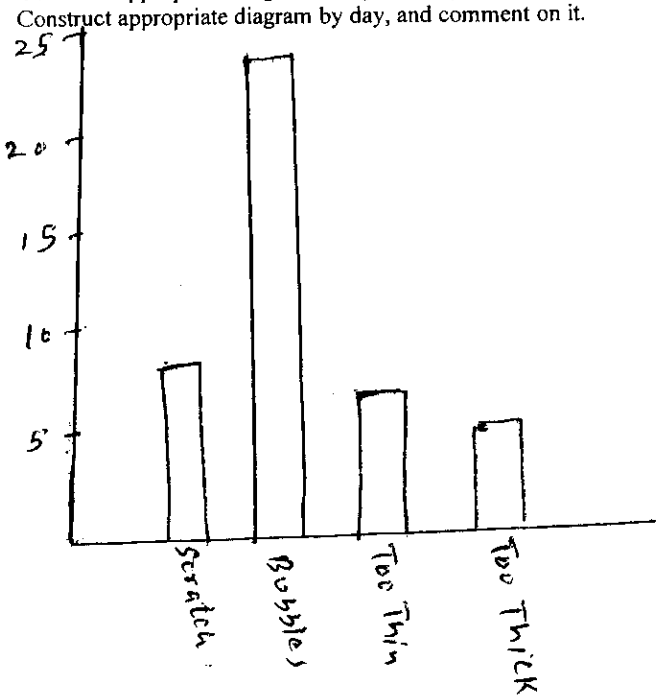
Q2. (2+2) =4.

The following data were obtained over a 3-day period on the frequency of occurrence of defects in a plastic part. Assume the number of parts involved is the same for each day.

Day	Type of Defect				
	Scratch	Bubbles	Too Thin	Too Thick	
1	2	8	4	1	15
2	0	7	2	3	12
3	6	9	0	1	16
	8	24	6	5	

a. Construct appropriate diagram using defect type as the variable, and comment on it.
b. Construct appropriate diagram by day, and comment on it.

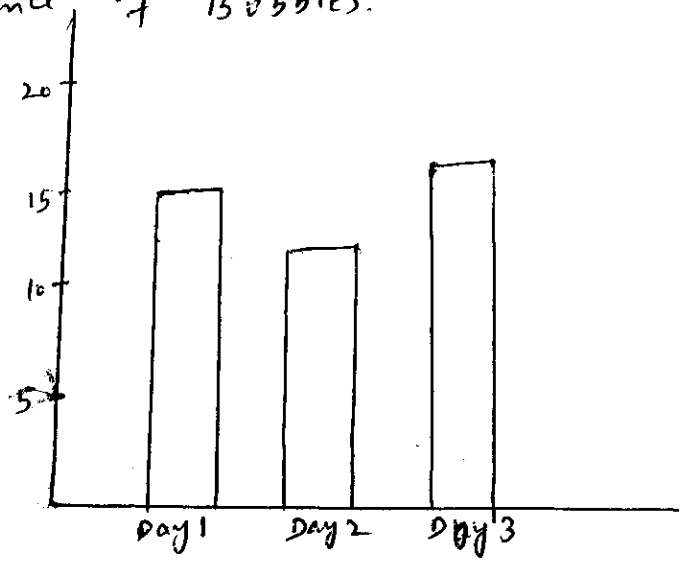
a)



(1)

The most frequent type of defect is occurence of Bubbles. (1)

b)



(1)

There is not much difference in number of defects from day to day. (1)

Q3. (3+3)=6

The probability that a computer contains a virus is 0.15, and the probability that it has a worm is 0.05. Also 17% computers have either a virus or a worm.

a. If somebody scans the computer and finds no virus, what is the probability that the computer contains a worm.
b. Are the events 'having a virus' and 'having a worm' independent? Give a probabilistic justification.

Let V: The computer contains a virus; P(V)= 0.15
Let W: The computer contains a worm; P(W)=0.05

$P(V \cup W) = 0.17$

a. $P(W \cap V) = P(W) + P(V) - P(W \cup V) = .05 + 0.15 - 0.17 = 0.03$ (1 point)

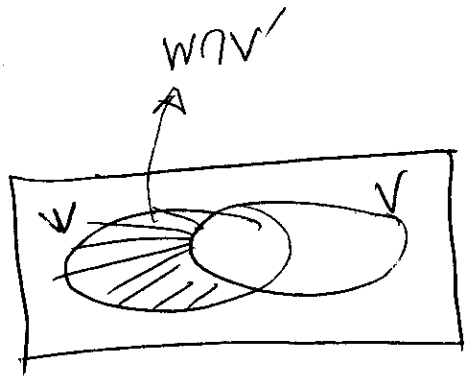
$P(W|V') = \frac{P(W \cap V')}{V'} = \frac{P(W) - P(W \cap V)}{1 - P(V)} = \frac{0.05 - .03}{1 - .15} = 0.0235$ (2 points)

b. No, since

$P(W \cap V) = 0.03 \neq P(V) * P(W) = 0.15 * 0.05 = 0.0075$

Or

$P(W|V) = \frac{P(W \cap V)}{P(V)} = \frac{0.03}{0.15} = 0.2 \neq P(W) = 0.05$



Q4. (3+3) = 6

Each of 2 identical cabinets has 2 drawers. Cabinet *A* contains a silver coin in each drawer, and cabinet *B* contains a silver coin in one of its drawers and a gold coin in the other. Suppose that the selection of cabinets is equally likely.

a. One cabinet is randomly selected, one of its drawers is opened, what is the probability that a silver coin is found?
b. If a silver coin is found in the first drawer, what is the probability that there is a silver coin in the other drawer?

Solution: Let S represents Silver Then

a) P(S)=P(S/A)P(A)+ P(S/B)P(B) = 1(1/2)+(1/2)(1/2)=0.75

(b) $P\{\text{silver in other} \mid \text{silver found}\}$

$$= \frac{P\{S \text{ in other. } S \text{ found}\}}{P\{S \text{ found}\}}.$$

To compute these probabilities. condition on the cabinet selected.

$$= \frac{1/2}{P\{S \text{ found} \mid A\}\,1/2 + P\{S \text{ found} \mid B\}\,1/2}$$

$$= \frac{1}{1+1/2} = \frac{2}{3}.$$

5. (2+3) = 5

A survey of those using a particular statistical software system indicated that 10% were dissatisfied. Half of those dissatisfied purchased the system from company A. It is also known that 20% of those surveyed purchased from company A.

a. What is probability of a satisfied customer purchasing the system from company A?
b. Given that the software package was purchased from company A, what is the probability that a particular user is dissatisfied?

Solution: We have $P(D) = 0.10$, $P(A|D) = 0.50$, $P(A) = 0.20$.

a. $P(A|S) = \frac{P(SA)}{P(S)}$?

Since $P(A) = P(AD) + P(AS)$,

or, $0.20 = P(D)\,P(A|D) + P(AS)$,

or, $0.20 = 0.10\,(0.50) + P(AS)$, → (1)

or, $P(AS) = 0.15$,

Also $P(S) = 1 - P(D) = 1 - 0.10 = 0.90$

$\therefore P(A|S) = \frac{0.15}{0.90} \approx 0.17$ → (1)

b. $P(D|A) = \frac{P(DA)}{P(A)} = \frac{0.05}{0.20} \approx 0.25$ → (1) → (1) → (1)