KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS
Term 112

STAT 302: STATISTICAL INFERENCE

FINAL EXAM   Saturday May 26, 2012

Name: ____________________   ID #: ____________________

☺ Important Notes:

- Write proper Mathematics
- State all your hypotheses, decisions and conclusions
- Show all your work, intermediate steps and final answer.
- Give reasons; clearly state any results you are using.

Question No	Full Marks	Marks Obtained
1	11	
2	5	
3	7	
4	10	
5	8	
6	6	
7	10	
8	8	
Total	55	

1) A large insurance company that hires many workers with disabilities wants to determine whether their disabilities affect such workers' performance. The following data were obtained:

Disability	Performance		
	Above Average	Average	Below Average
Blind	21	64	17
Deaf	16	49	14
No disability	29	93	28

At the 5% significance level decide whether it is reasonable to conclude that the disabilities have no effect on the workers' performance. State all your assumptions

2) Let $X_1, \ldots, X_n$ be a random sample from the exponential distribution

$$f(x) = e^{-(x-\theta)} I_{(\theta,\infty)}(x).$$

a) Find an unbiased estimator of θ.

b) Show that this estimator is consistent.

3) Based on a sample of size n, find the method of moments estimators of α and β, the parameters of a gamma distribution. You can use the fact that the mean and variance of the gamma distribution are $\alpha\beta$ and $\alpha\beta^2$ respectively.

4) Let $X_1,\ldots,X_n$ be a random sample from a distribution with probability function $f(x,\theta)$.

a) Explain the concept of maximum likelihood estimation.

b) Derive the maximum likelihood estimator of θ, the parameter of a Poisson distribution.

c) If the number of accidents that occur daily can be modeled as a Poisson random variable. The number of accidents was observed for 10 days and the results were as follows: {7, 3, 1, 2, 4, 1, 2, 3, 1, 2}. Find the maximum likelihood estimate of the probability of 0 or 1 accidents in one day. Justify your answer.

5) Let $X_1,\ldots,X_n$ be i.i.d random variables with density $f(x,\theta)$.

a) Define a sufficient statistic for θ.

b) If X is a Bernoulli random variable with $P(X=1)=\theta$, find (not just state) a sufficient statistic for θ.

c) Use the definition in a) to prove that the statistic you found in b) is sufficient.

6) Let $X_1,\ldots,X_n$ be a random sample from a normal distribution with unknown mean μ and unknown variance σ^2. Find an expression for the likelihood ratio statistic for testing

$$H_0:\sigma=\sigma_0 \quad \text{vs.} \quad H_a:\sigma\neq\sigma_0$$

7) Let $X_1,\ldots,X_n$ be a random sample from a uniform distribution over the interval $(0,\theta)$.

a) Find the most powerful level α test for testing $H_0:\theta=\theta_0$ against $H_a:\theta=\theta_a$, where $\theta_a<\theta_0$

b) Is the test in a) uniformly most powerful? Explain.

8) In a sequence of independent Bernoulli trial with success probability θ, let Y indicate the number of trials until the first success, i.e.

$$P(Y=y)=(1-\theta)^{y-1}\theta, \qquad y=1,2,\ldots$$

a) If θ has the following prior $f(\theta)=\frac{\Gamma(2\alpha)}{[\Gamma(\alpha)]^2}\left[\theta(1-\theta)\right]^{\alpha-1}$, find the posterior distribution of $\theta \mid y$.

b) Find the Bayes estimators of θ and $\theta(1-\theta)$