Department of Mathematics and Statistics
Semester 112

STAT302 Third Major Exam Sunday May 6, 2012

Name: ______________________ ID #: ________________

1) A company wants to know whether there is a significant difference in trainees recruited from two training facilities, A and B. Eleven trainees from facility A had a mean rating of 83.2, with a standard deviation of 11.4. Eight trainees from facility B had a mean rating of 86.2 with a standard deviation of 13.8.
   a) Test the hypothesis that two mean ratings are equal.
   b) Do you need any distributional assumptions? If yes, what?
   c) What do you require the samples to be?
   d) Are you making other assumptions? If yes, what? Justify statistically.

2) If $Y_1,\ldots,Y_n$ is a random sample from

$$f\left(y,\theta\right)=\theta^y(1-\theta)^{1-y}, \quad y=0,1$$

Suppose that we are interested in testing $H_0:\theta=\theta_0$ vs $H_a:\theta=\theta_a$, where $\theta_0<\theta_a$

Show that the most powerful test is equivalent to $\sum_{i=1}^{n} Y_i > k$, for some k.

3) Let $Y_1, \ldots, Y_n$ is a random sample from an exponential distribution with mean θ_1, and let $X_1, \ldots, X_m$ be a random sample from an exponential distribution with mean θ_2. Find the likelihood ratio test of $H_0 : \theta_1 = \theta_2 \quad vs \quad H_a : \theta_1 \neq \theta_2$

4) If $\overline{Y}$ is the mean of a random sample of size n from a normal population with unknown mean μ and known variance σ^2. The conjugate prior distribution for μ is a normal distribution with mean μ_0 and variance σ_0^2. The posterior distribution of μ has mean and variance given by: $\mu_1 = \dfrac{n\overline{Y}\sigma_0^2 + \mu_0\sigma^2}{n\sigma_0^2 + \sigma^2}$ and $\sigma_1^2 = \dfrac{\sigma_0^2\sigma^2}{n\sigma_0^2 + \sigma^2}$ respectively.

A distributor of soft drink vending machines feels that in a supermarket one of his machines will sell on the average 738 drinks per week. This mean will vary from market to market and the variation is measured by the standard deviation 13.4 drinks per week. So far as a machine is place in a particular market is concerned, the number of drinks sold will vary from week to week, and this variation is measured by the standard deviation 42.5 drinks per week. If one of the distributor's machines put into a new supermarket averaged 692 drinks per week over a period of 10 weeks. Assuming that all the variables involved are normal.

a) What is the probability that for this market the value of the mean is actually between 700 and 720 drinks per week?
b) Find a 95% credibility interval for the mean.