Department of Mathematics and Statistics
Semester 112

STAT302 First Major Exam Sunday February 19, 2012

Name: ______________________ ID #: ________________

1) If $Y_1, Y_2, \ldots, Y_n$ is a random sample from any continuous distribution with median *m*, what is $P\left(Y_{(n)} > m\right)$? *(5 pts.)*

2) Let *Y* be the time it takes a worker to complete a certain task, and assume that *Y* has the following density function

$$f(y) = \begin{cases} e^{-(y-\theta)} & y > \theta \\ 0 & otherwise \end{cases}; \quad \theta \text{ is a positive constant}$$

Let $Y_1, Y_2, \ldots, Y_n$ be a random sample of completion times. Find the expected minimum completion time. *(5 pts.)*

3) Let $Y_1, Y_2, \ldots, Y_5$ be a random sample from a normal population with mean 0 and variance 1, and let $\overline{Y} = (1/5)\sum_{i=1}^{5} Y_i$. Let Y_6 be another independent observation from the same population. Find the distribution of $2\left(5\overline{Y}^2 + Y_6^2\right) \Big/ \sum_{i=1}^{5}(Y_i - \overline{Y})^2$. Give details and reasons.

(5 pts.)

4) An economist wishes to estimate the average family income in a certain population. The population standard deviation is known to be \$4500, and the economist uses a random sample of size 225. What is the probability that the sample mean will fall within \$800 of the population mean? *(5 pts.)*

5) The service time in queues should not have a large variance; otherwise, the queue tends to build up. A bank regularly checks service time by its tellers to determine its variance. A random sample of 24 service times (in minutes) gives $s^2 = 8$.

a) Can you find an upper limit for the sample variance such that the probability that the sample variance does not exceed that upper limit is 0.90? *(4 pts.)*

b) What is the probability that the sample mean does not differ from the population mean by more than 1 minute? *(4 pts.)*

c) Do you need any assumptions to solve parts a) and b)? If yes, what? *(2 pts.)*