Math 654 (in Semester 112)

"Advanced Topics in Algebra"

by Jawad Abuhlail

Rationale/Objectives: The course provides the students with a geometric motivation and interpretations of a number of the basic results in commutative Algebra. On the other hand, it provides the students with the basic tools needed to begin studying Algebraic Geometry.

Text Book: David Eisenbud, *Commutative Algebra: with a View toward Algebraic Geometry* (Graduate Texts in Mathematics), Springer (2004).

Further Reading:

1) M. F. Atiyah and I. G. Macdonald, *Introduction to Commutative Algebra*, Addison-Wesley Publishing Co. (1969).

2) N. Bourbaki, *Commutative Algebra*, Chapters 1-7, Springer (1998).

3) L. Rowen, *Graduate Algebra: Commutative View*, Graduate Studies in Mathematics 73, AMS (2006).

Grading Policy:

Assignments	Midterm	Final
30%	30%	40%

Syllabus

Chapter	Section	Details	Weeks
Ch. 0		**Elementary Definitions**	**1**
	0.1	Rings and Ideals	
	0.2	Unique Factorization	
	0.3	Modules	
Ch. 1		**Roots of Commutative Algebra**	**5**
	1.1	Number Theory	
	1.2	Algebraic Curves and Function Theory	
	1.3	Invariant Theory	
	1.4	The Basis Theorem	
	1.6	Algebra and Geometry: The Nullstellensatz	
	1.7	Geometric Invariant Theory	
	1.8	Projective Varieties	
	1.9	Hilbert Functions and Polynomials	
	1.10	Free Resolutions and the Syzygy Theorem	
Ass. 1	Exercises	Noetherian Rings and Modules	
Ass. 2	Exercises	Algebra and Geometry	
Ass. 3	Exercises	Free Resolutions	
Ass. 4	Exercises	Spec, max-Spec and the Zariski Topology	
		MID TERM EXAM (30 %)	
Ch. 2		**Localization**	**3**
	2.1	Fractions	
	2.2	Hom and Tensor	
	2.3	The Construction of Primes	
	2.5	Products of Domains	
Ass. 5	Exercises	Constructing Primes	
Ass. 6	Exercises	Idempotents, Products and Connected Components	
Ch. 3		**Associated Primes and Primary Decomposition**	**6**
	3.1	Associated Primes	
	3.2	Prime Avoidance	
	3.3	Primary Decomposition	
	3.4	Primary Decomposition and Factoriality	
	3.6	Extracting Information from Primary Decomposition	
	3.7	Why is Primary Decomposition not Unique?	
	3.8	Geometric Interpretation of Primary Decomposition	
Ass. 7	Exercises	Total Quotients	
Ass. 8	Exercises	Prime Avoidance	
		FINAL EXAM (40 %)	

Midterm Exam (due April 1, 2012)

- Solve 12 problems from:

1.2, 1.3, 1.6, 1.8, 1.9, 1.10, 1.12, 1.13, 1.15, 1.16, 1.18, 1.19, 1.21, 1.22, 1.23, 1.24, 1.25

- Solve 3 problems from:

1.4, 1.5, 1.7, 1.11, 1.14, 1.17, 1.20

Final Exam (due May 20, 2012)

- Solve 12 problems of:

2.4, 2.5, 2.12, 2.13, 2,21, 2.24, 2.25

3.1, 3.4, 3.10, 3.12, 3.15, 3.16, 3.17, 3.18, 3.19. 3.20

- Solve 8 problems from the following (including at least 3 from Chapter 3):

2.2, 2.3, 2.6, 2.7, 2.8, 2,9, 2.10, 2.12, 2.19, 2.20, 2.22, 2.23, 2.26, 2.27

3.2, 3.3, 3.6, 3.7, 3.8, 3.9, 3.11, 3.14