Department of Mathematics and Statistics
Math 621 Topology-II
Semester 112
Midterm Exam
Due: April 1ˢᵗ 2012

1. Show that every compact metrizable space has a countable basis.

2. Show that Tietze extension theorem implies Urysohn's lemma.

3. Show that a second countable Hausdorff space need not be metrizable.

4. Let X be regular second countable space, and let U be an open set in X .

   (a) Show that U is a countable union of closed sets

   (b) Show that there exists a continuous function $f : X \to [0,1]$ such that $f(x) > 0$

      for all $x \varepsilon U$ and $f(x) = 0$ if $x \notin U$.

5. Let X be a compact Hausdorff space. Show that X is second countable iff X is
   metrizable.

6. Let X be locally compact Hausdorff space. If X is second countable show that X is
   metrizable. Show that the converse is not true.

7. Show that $I \times I$ in the dictionary ordered topology is not metrizable.