King Fahd University of Petroleum & Minerals
Department of Math. & Stat.

Math 568 - Final Exam (112)   Time: 2 hours 30 mns

Tuesday, May 22, 2012

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Name: ID #

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Problem 1	/10
Problem 2	/10
Problem 3	/5
Problem 4	/5
Problem 5	/15
Total	/45

Problem # 1. (10 marks) Given the one-dimensional problem

$$\begin{array}{ll} u_{tt}(x,t) - u_{xx}(x,t) = 0, \qquad x > 0, \quad t > 0 & \\ u(x,0) = \phi(x), \qquad u_t(x,0) = \psi(x), \qquad x \geq 0 & \qquad (I) \\ u_x(0,t) - u_t(0,t) = 2t, \qquad t \geq 0 & \end{array}$$

where ϕ and ψ are $C^2([0,+\infty))$, with $\phi(0) = \psi(0) = 0$.

a. Find a continuous solution for (I)

b. If $\phi(x) = x^2$ and $\psi(x) = 0$, write down the solution.

Problem # 2. (10 marks) Solve the Cauchy problem

$$\begin{aligned} &u_{tt} - (u_{xx} + u_{yy}) = 0, \qquad (x, y) \in \mathbb{R}^2, \quad t > 0 \\ &u(x, y, 0) = u_t(x, y, 0) = xy, \qquad (x, y) \in \mathbb{R}^2 \end{aligned}$$

Problem # 3. (5 marks) Assume that u is a solution of

$$u_{xx} + u_{yy} = 0, \qquad (x, y) \in \Omega$$

where Ω and the boundary conditions are shown if the figure.
Show that

$$0 \leq u(x, y) \leq 4, \qquad \forall (x, y) \in \Omega$$

Problem # 4. (5 marks) Consider

$$\begin{aligned} -\Delta u + u^3 &= f \qquad \text{in } \Omega \\ u &= \varphi \qquad \text{on } \partial\Omega \end{aligned} \tag{*}$$

where Ω is a bounded domain with a smooth boundary.
a. Show

$$(s^3 - r^3)(s - r) \geq 0, \qquad \forall s, r \in \mathbb{R}$$

b. Show that (*) has at most one solution.

Problem # 5. (15 marks) Given the problem

$$\begin{array}{ll} u_t(x,t) - u_{xx}(x,t) + 2u(x,t) = 0, \quad x \in (0,\pi), \quad t > 0 & \\ u(x,0) = \sin^2 x - 4\cos x, \quad x \geq 0 & (II) \\ u_x(0,t) = u_x(\pi,t) = 0, \quad t \geq 0 & \end{array}$$

a. Let $v(x,t) = u(x,t)e^{2t}$. Show that v satisfies

$$\begin{array}{ll} v_t(x,t) - v_{xx}(x,t) = 0, \quad x \in (0,\pi), \quad t > 0 & \\ v(x,0) = \sin^2 x - 4\cos x, \quad x \geq 0 & (III) \\ v_x(0,t) = v_x(\pi,t) = 0, \quad t \geq 0 & \end{array}$$

b. Solve (III) *c.* Show that there exists a constant C such that

$$|u(x,t)| \leq Ce^{-2t}$$