King Fahd University of Petroleum & Minerals
Department of Math. & Stat.

Math 568 - Midterm Exam (112)
Time: 2 hours 30 mns

Wednesday, April 11, 2012

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Name: ID #

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Problem 1	/10
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Problem 2	/10
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Problem 3	/10
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Problem 4	/10
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Problem 5	/10
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Total	/50

Problem # 1. (10 marks) Given

$$xu_x + yu_y + zu_z = 2u$$
$$u(x, y, z) = 1, \quad \forall(x, y, z) \in \Sigma,$$

where

$$\Sigma = \left\{ (x, y, z) \in \mathbb{R}^3 \ / \ z = \sqrt{1 - x^2 - y^2} \right\}$$

a) Show that Σ is noncharacteristic

b) Solve the problem

Problem # 2. (10 marks) By using the characteristic method, find two solutions for the problem

$$xu_x + yu_y + \tfrac{1}{2}(u_x^2 + u_y^2) - u = 0$$
$$u(x, 0) = \tfrac{1}{2}(1 - x^2)$$

Problem # 3. (10 marks) Given the 2nd-order PDE

$$4u_{xx} + x^2 u_{yy} - \frac{4}{x} u_x = 0, \qquad x > 0$$

a. Show, by a convenient change of variable, that the equation can be reduced to

$$w_{\xi\xi} + w_{\eta\eta} = 0$$

b. Solve the equation associated with the Cauchy data

$$u(x, 0) = \frac{x^2}{2}, \qquad u_y(x, 0) = 2$$

Problem # 4. (10 marks) Given the PDE

$$y^2 u_{xx} - 2xy u_{xy} + x^2 u_{yy} + y^2 u_x - (xy + y + \frac{x^2}{y}) u_y = 0, \qquad x > 0, \quad y > 0 \qquad \text{(P)}$$

a. Use an appropriate nonsingular transformation to reduce (P) to

$$w_\eta + w_{\eta\eta} = 0 \qquad \qquad (0.1)$$

b. Find a general solution of (P)

Problem # 5. (10 marks) In the domain

$$\Omega = \left\{ (x, y) \in \mathbb{R}^2 \ / \ 0 < x < 1, \ 0 < y < 1 \right\},$$

let

$$u(x, y) = \begin{cases} x & x \geq y \\ y & y > x \end{cases}$$

a) Show that $u \in C(\Omega)$ but $u \notin C^1(\Omega)$

b) Show that u is a weak solution for the equation

$$u_x + u_y = 1$$