Math 550 QUIZ #1 Name:..ID:..................

1. Give the definitions of each of the following concepts,

   - a field F

   - a vector space V over a field F

   - a subspace W of a vector space V

2. Let V be a vector space over a field F and $\{V_\alpha\}$ be a collection of subspaces of V. Show that $\cap_\alpha V_\alpha$ is a subspace of V.

3. Let $W_1, \cdots, W_n$ be subsets of a vector space V. Is $W = \sum_{k=1}^{n} W_k$ a subspace of V ?

4. Let W_1 and W_2 be subspaces of a vector space V such that $V = W_1 + W_2$ and $W_1 \cap V_2 = \{0\}$.

   Prove that for each vector v in V, there are unique vectors $v_1 \in W_1$ and $v_2 \in W_2$ such that $v = v_1 + v_2$.