KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
Department of Mathematics and Statistics

S112
MATH 550
Instructor: Dr. B. Chanane
EXAM I (Duration 90 Minutes)

Name:..ID#.............................

Exercise 1 *Let* T:$\mathbb{R}^3 \longrightarrow \mathbb{R}^2$ *given by*

$$T(x, y, z) = (x - 3y, x + 2y + z)$$

Let S_3 *and* S_2 *be the standard bases for* $\mathbb{R}^3$ *and* $\mathbb{R}^2$ *respectively, and let* $B_3 = \{(2,0,0),(0,1,3),(0,-1,0)\}$ *and* $B_2 = \{(2,1),(-1,1)\}$ *be some ordered bases for* $\mathbb{R}^3$ *and* $\mathbb{R}^2$ *respectively.*

(i) Show that T *is a linear transformation.*

(ii) Find the transition matrix P *from* B_2 *to* S_2*.*

(iii) Find the transition matrix Q *from* B_3 *to* S_3*.*

(iv) Find the matrix A_1 *representing* T *relative to the bases* S_3 *and* S_2*.*

(v) Find the matrix A_2 *representing* T *relative to the bases* B_3 *and* B_2*.*

(vi) How are the matrices A_1 *and* A_2 *related through* P *and* Q *?*

Exercise 2 *Let* W *be the solution space of*

$$\begin{cases} x + 2y - 2z = 0 \\ -x + y + 6z = 0 \\ 3x + 3y - 10z = 0 \end{cases}$$

(i) Find a basis B *of* W*.*

(ii) Extend B to a basis for $V = \mathbb{R}^3$.

(iii) Find a dual basis for V^*.

Exercise 3 *Let T be a linear operator on the vector space $\mathbb{R}^3$ which is represented in the standard ordered basis by the matrix*

$$A = \begin{pmatrix} 3 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$$

(i) Prove that T is diagonalizable by exhibiting a basis for $\mathbb{R}^3$, each vector of which is a characteristic vector of T.

(ii) Write A as a linear combination of appropriate projections E_1,E_2.