King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 536 [Functional Analysis II]
Second Semester 2011-2012 (112)

Exam II April 28, 2012 Time: 2 hours

Q1. (a) Let Pbe an orthogonal projection on an inner product space X. If $N\ (P)$ is the null space of P and $R\ (P)$ is the range of P, then show that $N(P) = [R(p)]^{\perp}$.

(b) If P is a bounded linear projection on a Hilbert space H, then show that P is self-adjoint and idempotent.

Q2. (a) Let P_1 and P_2 be projections on a Hilbert space$H$. If $P = P_1 + P_2$ is a projection, then prove that P projects H onto $Y = Y_1 \oplus Y_2$ where $P_1(H) = Y_1$ and $P_2\ (H) = Y_2$.

(b) Let K be a nonempty closed convex subset of a Hilbert space H. For $x \in H, z \in K$ is a projection of x if and only if $< x - z, y - z > \leq 0$ for all $y \in K$. Use this fact to prove that the projection operator $P_K(x)$ of H onto K satisfies $\| P_K(u) - P_K(V) \| \leq \| u - V \|$ for all $u, V \in H$.

Q3. (a) Let $\{T_n\}$ be a sequence of compact linear operators from a normed space X into a Banach space Y. If $\{T_n\}$ is uniformly operator convergent to T (i.e.$\| T_n - T \| \to 0$ as $n \to \infty$), then prove that limit operator T is compact.

(b) Let $T: l_2 \to l_2$ be defined by

$$T(x) = \frac{\xi_j}{j} \text{ where } x = \{\xi_j\} \epsilon l_2.$$

Use above part (a) to show that T is a compact operator.

Q4. (a) Let X and Y be normed spaces and $T \epsilon BL(X, Y)$, the space of all bounded linear operators from X into Y. Define Banach adjoint T^X of T from Y^* to X^*. Use an appropriate consequences of the Hahn – Banach theorem to prove that $\| T^X \| = \| T \|$.

(b) Let X and Y, be normed spaces. If T is a compact linear operator from X into Y, then verify that $T^{\times}$ is also compact.