King Fahd Univ. of Petroleum and Minerals
Faculty of Sciences
Department of Mathematics and Statistics

FINAL EXAM
(MATH. 465-112)

Name:
ID:

Prob. 1

Show that the zero solution of $y' = -y + y^2$ is asymptotically stable. Sketch all solutions in the (t, y) space. Sketch all solutions in the phase space.

Prob. 2

Show that for the linear system $y' = A(t)y$, where $A(t)$ is continuous for $0 \leq t < \infty$, stability or asymptotic stability of the zero solution implies that for of every solution.

Prob. 3

Determine for which of the following scalar equations the zero solution is stable, asymptotically stable or unstable

(a) $u'' + 2k^2u' + \alpha^2 u = 0$,

(b) $u^{(4)} - 2u'' + u = 0$.

Prob. 4

Use the Routh-Hurwitz criterion to derive a criterion for the asymptotic stability of the zero solution of $u^{(4)} + pu''' + qu'' + ru' + su = 0$.

Prob. 5

Show that if $\lambda(t)$ is a continuous nonnegative function on $0 \leq t < \infty$ such that $\lim_{t\to\infty} \lambda(t) = 0$, then $\lim_{t\to\infty} \int_t^{t+1} \lambda(s)ds = 0$.

Prob. 6

If $n = 2$ and $V(y_1, y_2) = \int_0^{y_1} g(\sigma)d\sigma + y_2^2/2$, where g is a continuous function and $\sigma g(\sigma) > 0$ for $\sigma \neq 0$, prove that V is positive definite on the

whole plane. What if $\sigma g(\sigma) \geq 0$?

Prob. 7

Analyze the simple pendulum equation $\theta'' + \sin\theta = 0$ in the neighborhood of $(0, 0)$. Sketch the phase portrait for $|y_1| < \pi$ and for $|y_1| < 2\pi$.

Prob. 8

Determine an estimate of the region of asymptotic stability in the phase plane for the equation $u'' + \varepsilon(1-u)^2 u' + u = 0$.