King Fahd Univ. of Petroleum and Minerals
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Department of Mathematics and Statistics

Major 1
(MATH. 465-112)

Name:
ID:

Prob. 1
Consider the system

$$\begin{cases} y_1' = y_1^2 \\ y_2' = y_1 + y_2 \end{cases}$$

(a) Check that $\phi_1(t) = \frac{\eta_1}{1-\eta_1(t-t_0)}$, $\phi_2(t) = \eta_2 e^{t-t_0} + \eta_1 \int_{t_0}^{t} \frac{e^{t-s}}{1-\eta_1(s-t_0)} ds$ is a solution for which $\phi_1(t_0) = \eta_1$, $\phi_2(t_0) = \eta_2$

(b) Discuss the interval of existence according to $\eta_1 > 0$, $\eta_1 = 0$, $\eta_1 < 0$.

Prob. 2
Consider the D.E.

$$y' = \begin{cases} 0, \ t \leq 0, \ -\infty < y < \infty \\ 2\sqrt{y}, \ t \geq 0, \ 0 \leq y < \infty \\ y^2, \ t \geq 0, \ -\infty < y < 0 \end{cases}$$

Determine whether $\phi(t) = \begin{cases} 1, \ t < 0 \\ (t+1)^2, \ t \geq 0 \end{cases}$ is a solution on $-\infty < t < \infty$.

Prob. 3
Show that $\phi(t) = -1/t$ is a solution of $y' = y^2$ passing through $(-1, 1)$ and it is the only solution passing through $(-1, 1)$. What is the largest interval on which it is a solution. What is your conclusion?

Prob. 4

Discuss the existence and uniqueness of solutions ϕ of $y''+p(t)y'+q(t)y = f(t)$, $\phi(t_0) = y_0$, $\phi'(t_0) = z_0$.

Prob. 5

(a) Show that $\phi(t) \equiv 0$ is the only solution of $y'' + p(t)y' + q(t)y = 0$, $\phi(0) = \phi'(0) = 0$, if p and q are continuous on some interval containing 0 in its interior.

(b) Show that if $\psi(t)$ is a solution of the D.E. $y''+p(t)y'+q(t)y = 0$ that is tangent to the t-axis at some point $(t_1, 0, 0)$ then $\psi(t) \equiv 0$.

Prob. 6

Find all continuous functions which are nonnegative on $0 \le t \le 1$ such that $f(t) \le \int_0^t f(s)ds$, $0 \le t \le 1$.