March 19,2012

Test♯1 Math311, sec 1

Net Time Allowed: 45 minutes

Name: **ID ♯ :** **Serial:**

Exercise1:(8 pts)

Let f be a continuous function on $\mathbb{R}$ and has the following properties:
(i) $\lim_{x \longrightarrow 0} f(x) = 1$ and $f(x_1 + x_2) = f(x_1)\, f(x_2),\ \ \forall\, x_1,\, x_2 \in \mathbb{R}$
Prove that:
a)- $f(x) > 0, \forall\, x \in \mathbb{R}$.
b)- $f(n\, x) = [f(x)]^n\ \forall\, n \in \mathbb{N}$.
c)- If $f(1) = 1$ then f is constant.

Exercise2:(06 pts)

Show that if f and g are uniformly continuous on $A \subseteq \mathbb{R}$ and if they are both bounded on A, then their product fg is uniformly continuous on $\mathbb{R}$.

Exercise3:(07 pts)
Let f be the Dirichlet function defined by:

$$f(x) = \begin{cases} 1 & if \quad x \quad is \quad rational \\ 0 & if \quad x \quad is \quad irrational \end{cases} \tag{1}$$

Show that f is not continuous at any point of $\mathbb{R}$.

Exercise4:(09 pts)
Let $I = [a,\, b]$ be a closed bounded interval and let $f : I \longrightarrow \mathbb{R}$ be continuous on I.
Prove that f is bounded on I.
(Hint: Use Bolzano-Weirstrass Theorem.)