April 11,2012

QUIZ♯2 Math311, sec 1

Net Time Allowed: 20 minutes

Name: **ID ♯ :** **Serial:**

Exercise 1:(05 pts)

Find 1) $\lim_{x \longrightarrow 1} x^{\frac{1}{1-x}}$

2) $\lim_{x \longrightarrow \infty} x^2 e^{-x}$

solution:

1) $\lim_{x \to 1} x^{\frac{1}{1-x}} \overset{0.5}{=} \lim_{x \to 1} \exp\left(\frac{\ln x}{1-x}\right)$ or $\lim_{x \to 1} \frac{\ln x}{1-x} = \lim_{x \to 1} \frac{\frac{1}{x}}{-1} = -1$ 01

Hence $\lim_{x \to 1} x^{\frac{1}{1-x}} \overset{01}{=} e^{-1}$

2) $\lim_{x \to \infty} x^2 e^{-x} \overset{01}{=} \lim_{x \to \infty} \frac{x^2}{e^x} \overset{0.5}{=} \lim_{x \to \infty} \frac{2x}{e^x} \overset{0.5}{=} \lim_{x \to \infty} \frac{2}{e^x} \overset{0.5}{=} 0$

Exercise 2:(05 pts)

Prove that $\frac{x-1}{x} < \ln x < x - 1$ for $x > 1$.

solution:

Let $f(x) = \ln x$, $x > 1$. Applying the mean Value theorem to f on $[1, x]$, 01 we get: $\exists c \in (1, x) \,/\, \ln x - \ln 1 = \frac{1}{c}(x-1)$ 01

or $1 < c < x \Rightarrow \frac{1}{x} \overset{01}{<} \frac{1}{c} < 1 \Rightarrow \frac{x-1}{x} \overset{01}{<} \frac{x-1}{c} < x - 1$

Hence $\frac{x-1}{x} \overset{01}{<} \ln x < x - 1$ for $x > 1$.