3. Show that:

$$1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}} \leq 2\sqrt{n}.$$

4. Show that:

$$2! \cdot 4! \cdot 6! \cdots (2n)! \geq ((n+1)!)^n.$$

5. Show that:

$$\sqrt{2 + \sqrt{2 + \sqrt{2 + \cdots + \sqrt{2}}}} = 2\cos\frac{\pi}{2^{n+1}},$$

where there are $n$ 2s in the expression on the left.

6. (Chebyshev Polynomials) Define $P_i(x)$ as follows:

$$\begin{aligned} P_0(x) &= 1 \\ P_1(x) &= x \\ P_{n+1}(x) &= xP_n(x) - P_{n-1}(x), \text{ for } n > 0. \end{aligned}$$

Show that

$$P_n(2\cos\theta) = \frac{\sin(n+1)\theta}{\sin\theta}.$$

7. Show that:

$$\sin\theta + \sin 2\theta + \sin 3\theta + \cdots + \sin n\theta = \frac{\sin\left(\frac{(n+1)\theta}{2}\right)\sin\left(\frac{n\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)}$$