Name: **ID ♯:** **section:**

Exercise1:

Let f be a continuous function and g, h two differentiable functions.

Find a formula for $\frac{d}{dx}\left(\int_{g(x)}^{h(x)} f(t)\,dt\right)$.

Solution: Let $\ell_1(x) = \int_a^{g(x)} f(t)\,dt$. Find what is $\ell_1'(x)$?

Let F be a primitive of f, That is $F'(t)=f(t)$, Now let $u=g(x)$, Then: $\int_a^{g(x)} f(t)\,dt = \int_a^{u} f(t)\,dt$.

By FTC1: $\frac{d}{du}\left(\int_a^{u} f(t)\,dt\right) = f(u)$; or $\frac{d}{dx}\left(\int_a^{g(x)} f(t)\,dt\right) = \frac{d}{du}\left(\int_a^{u} f(t)\,dt\right)\cdot\frac{du}{dx}$

So $\frac{d}{dx}\left(\int_a^{g(x)} f(t)\,dt\right) = f(g(x))\cdot g'(x)$

Then: $\boxed{\frac{d}{dx}\left(\int_{g(x)}^{h(x)} f(t)\,dt\right) = f(h(x))\cdot h'(x) - f(g(x))\cdot g'(x)}$

Exercise2:

Find the derivatives of the functions:

1. $f(x) = \int_{\sqrt{x}}^{x^3} \sin t\,dt$
2. $g(x) = \int_{\tan x}^{x^2} \cos t\,dt$

Solution:

1. Let $g(x)=\sqrt{x}$, $h(x)=x^3$,

$\Rightarrow g'(x)=\frac{1}{2\sqrt{x}}$, $h'(x)=3x^2$

$\Rightarrow \boxed{f'(x)=3\sin(x^3)\cdot x^2 - \frac{1}{2}\sin\sqrt{x}\cdot\frac{1}{\sqrt{x}} = 3x^2\sin x^3 - \frac{\sin\sqrt{x}}{2\sqrt{x}}}$

2. $g'(x) = 2\cdot\cos x^2\cdot x - \cos(\tan x)\cdot\sec^2 x$

$\Rightarrow \boxed{g'(x) = 2x\cos(x^2) - \cos(\tan x)\cdot\sec^2 x}$