King Fahd University of Petroleum and Minerals

Department of Mathematics and Statistics

Math 311
Exam II – 2012–2013 (112)
Thursday, April 19, 2012

Allowed Time: 120 minutes

Instructor: **Dr. Boubaker Smii**

Name: _____

ID #: _____

Section #: _____ Serial Number: _____

Instructions:

1. Write **clearly** and **legibly**. You may lose points for messy work.
2. **Show all your work**. No points for answers without justification.
3. **Calculators and Mobiles are not allowed.**

Question #	Grade	Maximum Points
1		**05**
2		**19**
3		**20**
4		**13**
5		**08**
Total:		**65**

Exercise 1:

Let $f : (0, 1] \longrightarrow \mathbb{R}$ such that $f(x) = \cos \frac{1}{x}$.
Show that f is continuous, but not uniformly continuous on $(0, 1]$.

Exercise 2: Prove that

1. $| \sin a - \sin b | \leq | a - b |, \quad \forall\, a, b \in \mathbb{R}.$
2. $\sin x \leq 1.2\, x \quad \forall\, x \geq 0.$
3. $\lim\limits_{x \longrightarrow 0} (e^{2x} + x)^{\frac{1}{x}} = e^3.$

Exercise 3: Prove that

a) $x - \frac{1}{6}x^3 \leq \sin x \leq x, \ \forall \ x \in [0, \pi]$

b) Use a) to find $\lim\limits_{x \longrightarrow 0} \dfrac{\sin x}{x}$

Exercise 4:

Find an upper bound for the magnitude of the error in the approximation

$$\ln x \approx (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}, \qquad |x-1| < \frac{1}{64}$$

Problem 5:

Use the definition to evaluate $I = \int_a^b e^t\, dt$.

Hint: You may use $1 + x + x^2 + ... + x^n = \frac{1-x^{n+1}}{1-x}$, $(x \neq 1)$