Math 302

Quiz 6

9/ 5/ 2012

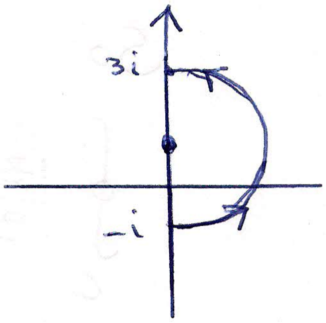
Name: **ID #**

==

Problem 1. (4 pts) Given the positively oriented right-half circle centered at i with radius 2

$$C : z = i + 2e^{it}, -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}.$$

Find $\int_C (\overline{z} + z) dz$.



$$I = \int_C (\overline{z} + z)\, dz$$

$$= \int_{-\pi/2}^{\pi/2} (\cancel{-i} + 2e^{-it} \cancel{+i} + 2e^{it})\, 2i e^{it}\, dt$$

$$= \int_{-\pi/2}^{\pi/2} (4i + 4i e^{2it})\, dt = 4i \int_{-\pi/2}^{\pi/2} dt + 4i \int_{-\pi/2}^{\pi/2} e^{2it}\, dt$$

$$= 4i\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right) + \frac{4i}{2i} \left. e^{2it} \right]_{-\pi/2}^{\pi/2}$$

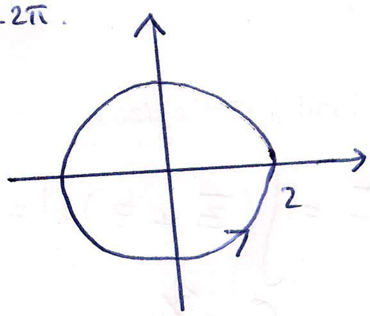
$$= 4\pi i + 2(\cancel{\cos\pi - \cos(-\pi)}) = 4\pi i$$

Problem 2. (3pts)
Compute

$$\oint_C \frac{|z|}{z}\,dz,$$

where C is the positively oriented circle centered at 0 with radius 2.

Remark. $|z|$ is not analytic, we cannot use Cauchy's integral formula. We have to parametrize C: $z = 2e^{it}$, $0 \le t \le 2\pi$.



So

$$\oint_C \frac{|z|}{z}\,dz = \int_0^{2\pi} \frac{2}{\cancel{2}\cancel{e^{it}}}\,\cancel{2}i\,\cancel{e^{it}}\,dt$$

$$= 2i\int_0^{2\pi} dt = 4\pi i$$

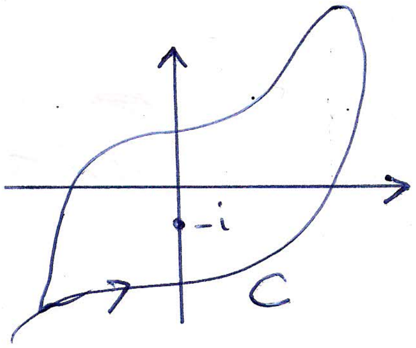
Problem 3. (3pts)
Compute

$$I = \oint_C \left(\frac{z^3 - i}{(z+i)^2} + e^{-z^2} \sin z \right) dz,$$

where C is shown in the figure.

$e^{-z^2} \sin z$ is analytic. So

$\oint_C (e^{-z^2} \sin z)\, dz = 0$ by Cauchy.



Thus,

$$I = \oint_C \frac{z^3 - i}{(z+i)^2}\, dz = \frac{2\pi i}{1!} \frac{d}{dz}(z^3 - i)\Big|_{z=-i}$$

$$= 2\pi i (3z^2)\Big|_{z=-i}$$

$$= 2\pi i (-3) = -6\pi i$$