Math 302

Quiz 4

9/ 4/ 2012

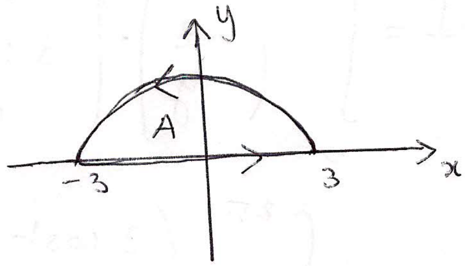
Name: **ID #**

Problem 1. (5 pts) Given a field

$$F = -y^3 \vec{i} + x^3 \vec{j}$$

Compute the line integral $\oint_C F.dr$, where C is the positively oriented ~~circle cen-~~ half circle tered at the origin with radius $= 3$, together with the segment $[-3, 3]$.

$$\oint_C F.dr = \iint_A \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

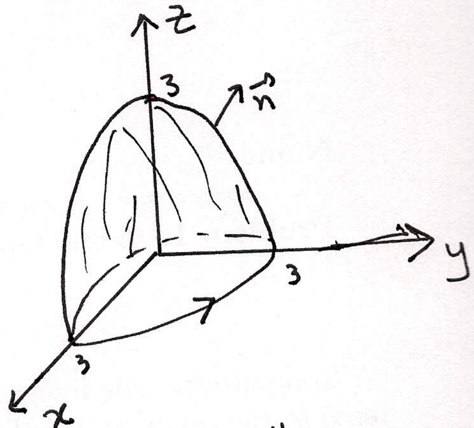


$$= \iint_A (3x^2 + 3y^2)\, dA$$

$$= \int_0^{\pi}\int_0^3 3r^2\, r\,dr\,d\theta = 3\int_0^{\pi}\int_0^3 r^3\,dr\,d\theta$$

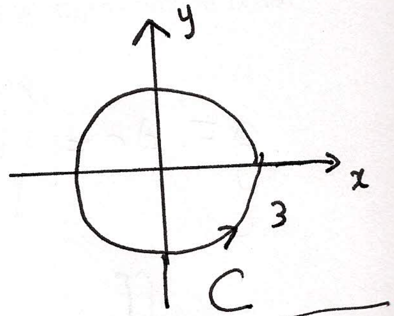
$$= 3\pi \left.\frac{r^4}{4}\right|_0^3 = \frac{243\pi}{4}.$$

Problem 2 (5pts): Let S be the paraboloid $z = 9 - x^2 - y^2$, for $z \geq 0$ and let C be its intersection with the xy-plane. Suppose that C is oriented so that the normal to S is pointing up. If $F(x, y, z) = x\,\vec{i} - z\,\vec{j} + y\,\vec{k}$, find $\int\int_S curlF \cdot ndS$,



$$I = \iint_S \operatorname{curl} F \cdot \vec{n}\, dS = \oint_C F \cdot dr$$

$$C = \{(3\cos t, 3\sin t, 0),\ 0 \leq t \leq 2\pi\}$$



$\boxed{z=0}$

$$I = \int_0^{2\pi} \begin{pmatrix} x \\ -z \\ y \end{pmatrix} \cdot \begin{pmatrix} -3\sin t \\ 3\cos t \\ 0 \end{pmatrix} dt$$

$$= \int_0^{2\pi} \begin{pmatrix} 3\cos t \\ 0 \\ 3\sin t \end{pmatrix} \cdot \begin{pmatrix} -3\sin t \\ 3\cos t \\ 0 \end{pmatrix} dt$$

$$= -9 \int_0^{2\pi} \sin t \cos t \, dt = -\frac{9}{2} \left. \frac{\sin^2 t}{} \right]_0^{2\pi} = 0$$