Math 302

Quiz 2

27/ 2/ 2012

Name: **ID #**

Problem 1 (5 points): Use the Gauss-Jordan method to find the inverse of

$$A = \begin{pmatrix} -2 & 0 & 1 \\ 3 & 1 & -1 \\ -1 & -1 & 2 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} -2 & 0 & 1 & 1 & 0 & 0 \\ 3 & 1 & -1 & 0 & 1 & 0 \\ -1 & -1 & 2 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R_1 \leftrightarrow -R_3} \left(\begin{array}{ccc|ccc} 1 & 1 & -2 & 0 & 0 & -1 \\ 3 & 1 & -1 & 0 & 1 & 0 \\ -2 & 0 & 1 & 1 & 0 & 0 \end{array}\right)$$

$$\xrightarrow[R_3+2R_1]{R_2-3R_1} \left(\begin{array}{ccc|ccc} 1 & 1 & -2 & 0 & 0 & -1 \\ 0 & -2 & 5 & 0 & 1 & 3 \\ 0 & 2 & -3 & 1 & 0 & -2 \end{array}\right) \xrightarrow[R_3+R_2]{2R_1+R_2} \left(\begin{array}{ccc|ccc} 2 & 0 & 1 & 0 & 1 & 1 \\ 0 & -2 & 5 & 0 & 1 & 3 \\ 0 & 0 & 2 & 1 & 1 & 1 \end{array}\right)$$

$$\xrightarrow[R_3/2]{R_1/2,\ R_2/-2} \left(\begin{array}{ccc|ccc} 1 & 0 & 1/2 & 0 & 1/2 & 1/2 \\ 0 & 1 & -5/2 & 0 & -1/2 & -3/2 \\ 0 & 0 & 1 & 1/2 & 1/2 & 1/2 \end{array}\right)$$

$$\xrightarrow[R_2+\frac{5}{2}R_3]{R_1-\frac{1}{2}R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/4 & 1/4 & 1/4 \\ 0 & 1 & 0 & 5/4 & 3/4 & -1/4 \\ 0 & 0 & 1 & 1/2 & 1/2 & 1/2 \end{array}\right)$$

So $$A^{-1} = \begin{pmatrix} -1/4 & 1/4 & 1/4 \\ 5/4 & 3/4 & -1/4 \\ 1/2 & 1/2 & 1/2 \end{pmatrix}$$

Problem 2 (5 points):Find all the eigenvalues of the matrix

$$B = \begin{pmatrix} 2 & 0 & 1 \\ 3 & 1 & -1 \\ -1 & -1 & 2 \end{pmatrix}$$

Choose one eigenvalue and compute a corresponding eigenvector.

Eigenvalues

$$\begin{vmatrix} 2-\lambda & 0 & 1 \\ 3 & 1-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} = (2-\lambda)\begin{vmatrix} 1-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + \begin{vmatrix} 3 & 1-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (2-\lambda)\left[\lambda^2 - 3\lambda + 2 - 1\right] + \left[-3 + 1 - \lambda\right]$$

$$= (2-\lambda)(\lambda^2 - 3\lambda + 1) - (2+\lambda)$$

$$= 2\lambda^2 - 6\lambda + \cancel{2} - \lambda^3 + 3\lambda^2 - \lambda - \cancel{2} - \lambda$$

$$= -\lambda^3 + 5\lambda^2 - 8\lambda = -\lambda(\lambda^2 - 5\lambda + 8) = 0$$

$\therefore \lambda = 0$ or $\lambda = \dfrac{5 \pm i\sqrt{7}}{2}$ are the eigenvalues.

An Eigenvector $\boxed{\lambda = 0}$

$$\begin{pmatrix} 2 & 0 & 1 \\ 3 & 1 & -1 \\ -1 & -1 & 2 \end{pmatrix} \xrightarrow{-R_3 \leftrightarrow R_1} \begin{pmatrix} 1 & 1 & -2 \\ 3 & 1 & -1 \\ 2 & 0 & 1 \end{pmatrix} \xrightarrow[R_3 - 2R_1]{R_2 - 3R_1} \begin{pmatrix} 1 & 1 & -2 \\ 0 & -2 & 5 \\ 0 & -2 & 5 \end{pmatrix}$$

$$\xrightarrow[R_3 - R_2]{-R_2/2} \begin{pmatrix} 1 & 1 & -2 \\ 0 & 1 & -5/2 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{R_1 - R_2} \begin{pmatrix} 1 & 0 & 1/2 \\ 0 & 1 & -5/2 \\ 0 & 0 & 0 \end{pmatrix}.$$

$$\Rightarrow \begin{array}{l} x_1 = -\frac{1}{2} x_3 \\ x_2 = \frac{5}{2} x_3 \end{array} \Longrightarrow E_1 = \begin{pmatrix} -1 \\ 5 \\ 2 \end{pmatrix} \text{ is an eigenvector.}$$