KFUPM – Department of Mathematics and Statistics – Term 112

MATH 280

Exam 3 (Tuesday, May 01, 2012)

Duration: 2 Hours

NAME:________________________ ID:________________________ Section: _01_

Exercise 1 (10 points)

Let $V = R^3$ be the Euclidean space (with the standard inner product) and set $S = \{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \}$.

Apply Gram-Schimdt process to construct an orthonormal basis from the basis S.

Exercise 2 (20 points)
Let V and W be two n-dimensional vector spaces over the same field, and with bases respectively $S = \{v_1, ..., v_n\}$ and $T = \{w_1, ..., w_n\}$. Prove that there is a unique linear transformation $L : V \to W$ such that $L(v_i) = w_i$ for every $i = 1, 2, ..., n$.

Exercise 3 (20 points)

Let $V = \mathrm{M}_3(R)$ be the vector space of all 3×3 matrices with real coefficients and set $A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix}$. Let $L : V \to V$ be the linear transformation defined by $L(B) = AB$.

1- Find $KerL$ and a basis for $KerL$.

2- Find $Range(L)$ and a basis for $Range(L)$.

Exercise 4 (20 points)

Let $L : R^3 \rightarrow R^2$ be the linear transformation given by $L\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 2a+b \\ b+2c \end{pmatrix}$ and let S_1 and T_1 be the standard bases for R^3 and R^2 respectively. Set $S_2 = \{\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}\}$ and $T_2 = \{\begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \end{pmatrix}\}$.

1-Find the matrix $[L]_{S_1, T_1}$ representing L with respect to the bases S_1 and T_1.

2-Find the matrix $[L]_{S_2, T_2}$ representing L with respect to the bases S_2 and T_2.

3-Find the relation between $[L]_{S_1, T_1}$ and $[L]_{S_2, T_2}$

Exercise 5 (20 points)

Let $A=(a_{ij})$ and B be $n\times n$ matrices and α a real number.

1-Use the **definition of** $\det A$ in terms of permutations to prove that $\det \alpha A=\alpha^n \det A$.

2-Prove that $adj(A^T)=(adj(A))^T$

3-Prove that $ABadj(AB)=Aadj(A)Badj(B)$

4-Pove that if A and B are nonsingular matrices, then $adj(AB)=adj(A)adj(B)$

Exercise 6 (10 points)

Use the cofactor methods to find the inverse of the matrix $A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$