KFUPM – Department of Mathematics and Statistics – Term 112

MATH 280

Exam 1 (Duration = 90 minutes)

NAME:________________________ ID:________________________ Section: _01_

Exercise 1 (15 points)

1-Use the **augmented matrix and echelon form** to solve the following system.

$$(S)\ \begin{cases} \dfrac{dx}{dt} - \dfrac{dy}{dt} + \dfrac{dz}{dt} = 1 \\ \dfrac{dx}{dt} - \dfrac{dy}{dt} - \dfrac{dz}{dt} = -1 \\ \dfrac{dx}{dt} - 2\dfrac{dy}{dt} + 2\dfrac{dz}{dt} = 1 \end{cases}$$

2-Solve the boundary value Problem $x(0) = 0, y(0) = 1, z(0) = 2$

Exercise 2 (15 points)

1-Prove that every $n \times n$ matrix A can be written as $A = S + K$ where S is an $n \times n$ symmetric matrix and T is an $n \times n$ skew-symmetric matrix.

2-Application $A = \begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

Exercise 3 (15 points)

Consider a linear system (S) of the form $AX = Y$ where A is an $n \times n$ matrix and $Y \neq 0$. Let $X_1, X_2, ..., X_n$ be solutions of the system (S).

1-Verify that $X_1 + X_2$ is not a solution of the system (S).

2-Verify that $2X_1 - 2X_2 + X_3$ is a solution of the system (S).

3-Under which condition on $\alpha_1, \alpha_2, ..., \alpha_n$, $\alpha_1 X_1 + \alpha_2 X_2 + ... + \alpha_n X_n$ is a solution of the system (S).

Exercise 4 (15 points) Find all 3×3 matrices A such that the commutator $[A,...]=0$.
Notice that $[A,B]=AB-BA$ for any matrix B

Exercise 5 (20 points). Use elementary row operations to find the inverse of the matrix

$$A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \\ 2 & -1 & 1 \end{pmatrix}$$

Exercise 6 (20 points)

Determine whether W is a subspace of the given space V.

1- $V = R_3$ and $W = \{(a,b,c) \in R_3 | 2a - b + 2c = 0\}$

2-Find two vectors u,v such that $W = Span(u,v)$

3-$V = C(R)$ the vector space of all continuous function and W the subset of all continuous odd functions.