# Version 1

King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 260
Final Exam, Semester II, 2011-2012
Saturday May 26, 2012
Net Time Allowed: 180 minutes (7:30am-10:30am)

Name:______________________________

ID:______________Section:______________

| Q# | Marks | Maximum Marks |
|---|---|---|
| 1 | | 15 |
| 2 | | 15 |
| 3 | | 25 |
| 4 | | 15 |
| 5 | | 10 |
| 6 | | 15 |
| 7-15 | | 45 |
| Total | | 140 |

1. Write clearly.
2. Show all your steps.
3. No credit will be given to wrong steps.
4. Do not do messy work.
5. Calculators and mobile phones are NOT allowed in this exam.
6. Turn off your mobile.

**Note**:
For Part II you should write your answers in the box below.

**Part II**

| 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |

# Part I

1. Find the general solution of the differential equation

$$y'' + 9y = 2\cos 3x + 3\sin 3x.$$

2. Use the method of variation of parameters to find a particular solution of

$$(x^2 - 1)y'' - 2xy' + 2y = (x^2 - 1)^2.$$

Hint: ($y_1 = x$ and $y_2 = 1 + x^2$ form a fundamental set of solutions for the associated homogeneous differential equation.)

3. Let $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$.

(a) Show that the characteristic polynomial of $A$ is: $P_A(\lambda) = (\lambda - 1)(\lambda - 2)^2$

(b) Find the eigenvalues and bases for the corresponding eigenspaces of $A$.

(c) Is $A$ diagonalizable?

(d) Find the Jordan canonical form of $A$.

4. Solve the following system: $Y' = AY$, where

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix}.$$

5. Convert the following system of differential equations into a system of first order differential equations in matrix form.

$$x'' = 5x - 4y$$
$$y'' = -4x + 5y + \sin t$$

.

6. Let $\alpha, \beta$ be real numbers. Consider the initial value problem

$$y'' + \alpha y' + \beta y = 0, \qquad y(0) = 3,\ \ y'(0) = 5.$$

Suppose that the differential equation has a fundamental set of solutions $\{y_1, y_2\}$, with $y_1 = e^{-x}$ and the Wronskian $\omega(y_1, y_2) = 4e^{2x}$.

(a) Show that $e^{3x}$ is a solution to the differential equation.

(b) Determine $\alpha, \beta$.

(c) Solve the initial value problem.

## Part II

7. A tank initially contains 600 liters of salt water with concentration $\frac{1}{15}$ kg/L. Suppose that a solution of salt water with concentration $\frac{1}{5}$ kg/L flows into the tank at a rate of 25 L/min. The solution in the tank is kept uniform. The solution flows out of the tank at a rate of 50 L/min. The amount of a salt in the tank at the end of 18 minutes is equal to:

   (a) 20 Kg

   (b) 41 Kg

   (c) 35 Kg

   (d) 25 Kg

   (e) 36 Kg

8. Which statements among the following is TRUE?

   (a) $\{(x, y) \in \mathbb{R}^2 |\ xy = 1\}$ is a subspace of $\mathbb{R}^2$.

   (b) $\{(x, y) \in \mathbb{R}^2 |\ 2x - y = 1\}$ is a subspace of $\mathbb{R}^2$.

   (c) $\{(x, y) \in \mathbb{R}^2 |\ x - 2y = 0\}$ is a subspace of $\mathbb{R}^2$.

   (d) $\{(x, y) \in \mathbb{R}^2 |\ xy = 0\}$ is a subspace of $\mathbb{R}^2$.

   (e) $\{(x, y) \in \mathbb{R}^2 |\ x^2 = y^2\}$ is a subspace of $\mathbb{R}^2$.

9. Let $A$ be a $3 \times 3$ matrix such that $\text{rank}(A) = 1$. Then:

(a) $\dim(RS(A)) = 2$.

(b) $\dim(NS(A)) = 2$.

(c) $A$ is a nonsingular matrix.

(d) $\dim(CS(A)) = 2$.

(e) $\text{rank}(2A) = 2$.

10. Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 4 & 9 & 16 \\ 1 & 8 & 27 & 64 \end{bmatrix}$.

(a) $\det(A) = 12$.

(b) $\det(A) = 9$.

(c) $\det(A) = 8$.

(d) $\det(A) = 6$.

(e) $\det(A) = 5$.

11. Let $A$ be a $3 \times 3$ matrix such that $A^2 - 2A + 13I_3 = 0$. Then:

(a) $A$ is a nonsingular matrix.

(b) $\text{rank}(A) < 3$.

(c) The reduced echelon form of $A$ is $A_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

(d) There exists $\lambda \in \mathbb{R}$ such that $A = \lambda I_3$.

(e) $\det(A - 2I_3) = 0$.

12. Let $S = \{v_1, v_2, v_3\}$ where

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \; v_2 = \begin{bmatrix} 2 \\ 9 \\ 0 \end{bmatrix}, \; v_3 = \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}.$$

Then:

(a) $S$ is linearly independent but does not span $\mathbb{R}^3$.

(b) $S$ spans $\mathbb{R}^3$ and $S$ is linearly dependent.

(c) $S$ does not span $\mathbb{R}^3$ and $S$ is linearly dependent.

(d) $S$ forms a basis for $\mathbb{R}^3$.

(e) None of the above.

13. The solution of the linear first order differential equation

$$(x^2+1)\frac{dy}{dx}+3xy=6x, \quad y(0)=3.$$

at $x=2$ is:

(a) $2+(5)^{-3/2}$

(b) $2+2(5)^{3/2}$

(c) $0$

(d) $2+3(5)^{3/2}$

(e) $2-(5)^{-3/2}$

14. The value of the general solution at $x=1$ of the Bernoulli differential equation

$$\frac{dy}{dx}+(\frac{6}{x})y=3y^{4/3}$$

is:

(a) $y(1)=\frac{1}{C^3}$

(b) $y(1)=C^3$

(c) $y(1)=\frac{1}{(C-1)^3}$

(d) $y(1)=\frac{1}{(1+C)^3}$

(e) $0$

15. Given that $y_1 = x\cos(\ln x)$ is a solution of the differential equation

$$x^2y'' - xy' + 2y = 0.$$

The value of the second solution $y_2$ (Using reduction of order) at $x = e$ is qual to:

(a) 0

(b) $e\sin 1$

(c) $\pi e$

(d) 1

(e) $e\cos 1$