King Fahd University of Petroleum and Minerals Department of Mathematics and Statistics Math 260

Exam II, Semester II, 2011-2012 Thursday April 19, 2012

Net Time Allowed: 120 minutes (3:30pm-5:30pm)

Name:	 	 	
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Q#	Marks	Maximum Marks
1		10
2		15
3		12
4		10
5		20
6		7
7		6
8		10
9		10
Total		100

- 1. Write clearly.
- 2. Show all your steps.
- 3. No credit will be given to wrong steps.
- 4. Do not do messy work.
- 5. Calculators and mobile phones are NOT allowed in this exam.
- 6. Turn off your mobile.

Note:

For Part II you should write your answers in the box below.

Part II

a	b	С	d	е

Part I

1. Is $S = \{A_1, A_2, A_3, A_4\}$ a basis for $M_{2\times 2}$, where

$$A_1 = \begin{bmatrix} 3 & 6 \\ 3 & -6 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}, A_3 = \begin{bmatrix} 0 & -8 \\ -12 & -4 \end{bmatrix}, A_4 = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}.$$

2. Let
$$A = \begin{bmatrix} 1 & 1 & -1 & 2 & 0 \\ 1 & 2 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 2 \end{bmatrix}$$
.

- (a) Find a basis for NS(A).
- (b) Find the rank of A.

- 3. Consider the following differential equation: $y' + 2y^2 = y$.
 - (a) Determine the equilibrium solutions.
 - (b) On each region determined by the equilibrium solutions, decide whether the solutions are increasing or decreasing, and the associated curves are concave up or down.
 - (c) Sketch graphs of solutions to the given DE.

- 4. Consider the following differential equation: $y' = (1 + e^{-x})(y^2 1)$ (1)
 - (a) Find the equilibrium solutions of (1).
 - (b) Solve the initial value problem

$$y' = (1 + e^{-x})(y^2 - 1)$$

$$y(0) = 0.$$

- 5. Solve each of the following differential equations:
 - (a) $(\cos x + \ln y)dx + (\frac{x}{y} + e^y)dy = 0.$ (b) $y' + 2y = \sin x.$

6. Write the following Homogenous differential equation as a separable equation: (DO NOT SOLVE IT)

$$x\frac{dy}{dx} = y + \sqrt{x^2 - y^2}.$$

7. Write the following Bernoulli differential equation as a linear equation: (DO NOT SOLVE IT)

$$y' + xy = xy^3.$$



Part II

9. Answer TRUE $(\sqrt{})$ or FALSE (\times)

(10pts)

- (a) If A is a 3×5 matrix, then $\operatorname{rank}(A) \leq 3$.
- (b) The functions x^2 and x|x| are linearly independent over $(-\infty,\infty)$.
- (c) The order of the differential equation $\frac{d^7y}{dt^7} + \frac{d^5y}{dt^5} \left[\frac{d^4y}{dt^4} \right] + t^9 = \sin t$ is 9.
- (d) Any separable differential equation is exact.
- (e) Any linear differential equation is also a Bernoulli equation.