King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 260
Exam II, Semester II, 2011-2012
Thursday April 19, 2012
Net Time Allowed: 120 minutes (3:30pm-5:30pm)

Name:_____

ID:_____ Section:_____

Q#	Marks	Maximum Marks
1		10
2		15
3		12
4		10
5		20
6		7
7		6
8		10
9		10
Total		100

1. Write clearly.
2. Show all your steps.
3. No credit will be given to wrong steps.
4. Do not do messy work.
5. Calculators and mobile phones are NOT allowed in this exam.
6. Turn off your mobile.

Note:
For Part II you should write your answers in the box below.

Part II

a	b	c	d	e

Part I

1. Is $S = \{A_1, A_2, A_3, A_4\}$ a basis for $M_{2\times 2}$, where
$A_1 = \begin{bmatrix} 3 & 6 \\ 3 & -6 \end{bmatrix}$, $A_2 = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$, $A_3 = \begin{bmatrix} 0 & -8 \\ -12 & -4 \end{bmatrix}$, $A_4 = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$.

2. Let $A = \begin{bmatrix} 1 & 1 & -1 & 2 & 0 \\ 1 & 2 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 2 \end{bmatrix}$.

(a) Find a basis for $NS(A)$.

(b) Find the rank of A.

3. Consider the following differential equation: $y' + 2y^2 = y$.

   (a) Determine the equilibrium solutions.

   (b) On each region determined by the equilibrium solutions, decide whether the solutions are increasing or decreasing, and the associated curves are concave up or down.

   (c) Sketch graphs of solutions to the given DE.

4. Consider the following differential equation: $y' = (1 + e^{-x})(y^2 - 1)$ (1)

   (a) Find the equilibrium solutions of (1).

   (b) Solve the initial value problem

   $y' = (1 + e^{-x})(y^2 - 1)$
   $y(0) = 0.$

5. Solve each of the following differential equations:

   (a) $(\cos x + \ln y)dx + (\frac{x}{y} + e^y)dy = 0$.

   (b) $y' + 2y = \sin x$.

6. Write the following Homogenous differential equation as a separable equation: (DO NOT SOLVE IT)

$$x\frac{dy}{dx} = y + \sqrt{x^2 - y^2}.$$

7. Write the following Bernoulli differential equation as a linear equation: (DO NOT SOLVE IT)

$$y' + xy = xy^3.$$

8. A thermometer reading 70^oF is placed in an oven preheated to a constant temperature. If the thermometer reads 110^oF after $1/2$ a minute and 145^oF after 1 minute, then how hot is the oven?

Part II

9. Answer TRUE ($\surd$) or FALSE ($\times$) (10pts)

(a) If A is a 3×5 matrix, then $\text{rank}(A) \leq 3$.

(b) The functions x^2 and $x|x|$ are linearly independent over $(-\infty, \infty)$.

(c) The order of the differential equation $\frac{d^7y}{dt^7} + \frac{d^5y}{dt^5}[\frac{d^4y}{dt^4}] + t^9 = \sin t$ is 9.

(d) Any separable differential equation is exact.

(e) Any linear differential equation is also a Bernoulli equation.