King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 260
Exam I, Semester II, 2011-2012
Tuesday February 28, 2012
Net Time Allowed: 90 minutes (8:30pm-10:00pm)

Name:_____

ID:_____ Section:_____

Q#	Marks	Maximum Marks
1		5
2		5
3		6
4		5
5		4
6		5
7		4
8		4
9		6
10		6
Total		50

1. Write clearly.
2. Show all your steps.
3. No credit will be given to wrong steps.
4. Do not do messy work.
5. Calculators and mobile phones are NOT allowed in this exam.
6. Turn off your mobile.

1. Determine the value(s) of a so that the system

$$\begin{aligned} -x + y + 2z &= a^2 \\ x + 2y - z &= 2a \\ 2x + y - 3z &= 1 \end{aligned}$$

has a solution.

2. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Show that

$$A^2 = (a + d)A - (ad - bc)I_2,$$

where I_2 is the 2×2 identity matrix.

3. Find the inverse of the matrix A or determine that A^{-1} does not exist.

$$A = \begin{bmatrix} 0 & -2 & 1 \\ 2 & 4 & -1 \\ 2 & 1 & 2 \end{bmatrix}.$$

4. Evaluate $\begin{vmatrix} b+c & c+a & b+a \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$.

5. If A and B are 4×4 matrices with $|A| = 4$ and $|B| = 5$, find $|AB| - |2A^{-1}|$.

6. Write the matrix $A = \begin{bmatrix} 2 & 7 \\ 1 & 4 \end{bmatrix}$ as a product of elementary matrices.

7. What is the inverse of the following elementary matrix?

$$A = \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Note: You should think about elementary matrices.

8. Do $x^2 + 1, x^2 + x, x + 1$ span P_2?

9. Define two operations on $\mathbb{R}^2$ by:

$$(x, y) \oplus (a, b) = (x + a, y + b), \text{ for all } (x, y), (a, b) \in \mathbb{R}^2$$
$$\text{and } c \odot (x, y) = (x, cy) \text{ for all } c \in \mathbb{R} \text{ and } (x, y) \in \mathbb{R}^2.$$

Determine which properties of a vector space fail to hold for $(E, \oplus, \odot)$.

10. Let W be the set of all vectors $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ in $\mathbb{R}^4$ such that

$$x_1 + 2x_2 + 3x_3 + 4x_4 = 0.$$

Is W a vector subspace of $\mathbb{R}^4$?