MATH 202.18 (Term 102)

Quiz 6 (Chap. 8.1-8.3) Duration: 20mn

Name: ID number:

Solve the homogeneous system

$$X' = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 0 & -1 & 1 \end{pmatrix} X.$$

$$\det(A-\lambda I) = \begin{vmatrix} 1-\lambda & 1 & 1 \\ 1 & 2-\lambda & 0 \\ 0 & -1 & 1-\lambda \end{vmatrix}$$

$$= (1-\lambda)(2-\lambda)(1-\lambda) - (1-\lambda+1)$$

$$= (2-\lambda)\left[(1-\lambda)^2 - 1\right]$$

$$= (2-\lambda)(1-\lambda-1)(1-\lambda+1)$$

$$= -\lambda(2-\lambda)^2$$

$\lambda = 0$, $\lambda = 2$ (repeated eigenvalue of multiplicity 2)

$(A-\lambda I)K = 0$

$\boxed{\lambda=0}$ $(A-0I)K_1 = 0$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x + 2z = 0$, $x = -2t$

$y - z = 0$, $y = t$

$z = t$

$$K_1 \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$$

$\boxed{\lambda=2}$ $(A-2I)K_2 = 0$

$$\left(\begin{array}{ccc|c} -1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \to$$

$x = 0$

$y + z = 0$, $y = -t$

$z = t$

$$K_2 \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

Now, we solve $(A-2I)P = K_2$

$$\left(\begin{array}{ccc|c} -1 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & -1 & -1 & 1 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & 1 & -1 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x = -1$

$y + z = -1$

$z = t$

$$P \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$$

$\Rightarrow$ The general solution is

$$\boxed{X = c_1 \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} e^{2t} + c_3 \left[t \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} \right] e^{2t}}$$