MATH 202.10 (Term 102)

Quiz 2 (Chap. 2.3-3.1) Duration: 20mn

Name: ID number:

1.)(5pts)Solve the exact DE: $(\cos^2(x+y)+\frac{xy^2}{2})dx+(\frac{1}{2}\cos 2(x+y)+\frac{x^2y}{2})dy=0.$

2.)(5pts)Solve by substitution the DE: $\frac{dy}{dx}=\frac{\sqrt{-x+y}+2}{\sqrt{-x+y}}.$

Solution

1.) $M=\cos^2(x+y)+xy^2/2$

$N=\frac{1}{2}\cos 2(x+y)+\frac{x^2y}{2}$

$\Rightarrow M_y=-2\cos(x+y)\sin(x+y)+xy$

$=-\sin 2(x+y)+xy$

$\Rightarrow N_x=-\sin 2(x+y)+xy$

$M_y=N_x \Rightarrow$ DE is exact.

$\frac{\partial f}{\partial x}=\cos^2(x+y)+\frac{xy^2}{2}$; $\frac{\partial f}{\partial y}=\frac{1}{2}\cos 2(x+y)+\frac{x^2y}{2}$

$f(x,y)=\frac{\sin 2(x+y)}{4}+\frac{x^2y^2}{4}+h(x)$

$\frac{\partial f}{\partial x}=\frac{\cos 2(x+y)}{2}+\frac{xy^2}{2}+h'(x)=$

$\cos^2(x+y)+\frac{xy^2}{2}$.

Noting that $\cos^2(x+y)=\frac{\cos 2(x+y)+1}{2}$

$\Rightarrow h'(x)=\frac{1}{2}$.

$h(x)=x/2$.

$\Rightarrow \boxed{\frac{\sin 2(x+y)}{4}+\frac{xy^2}{2}+\frac{x}{2}=C}$

2.) $\frac{dy}{dx}=\frac{\sqrt{-x+y}+2}{\sqrt{-x+y}}$

let $u=-x+y$, $\frac{du}{dx}=-1+\frac{dy}{dx}$

$\Rightarrow \frac{du}{dx}+1=\frac{\sqrt{u}+2}{\sqrt{u}}$

$\Rightarrow \frac{du}{dx}=\frac{2}{\sqrt{u}} \Rightarrow \int\sqrt{u}\,du=2\int dx$

$\frac{2}{3}u^{3/2}=2x+C$

$\frac{2}{3}(-x+y)^{3/2}=2x+C$

$\boxed{(-x+y)^{3/2}=3x+C}$