MATH 202.18 (sem. 112)
Quiz 1 (Sects. 1.1-2.2) Duration: 20mn

Name: ID number:

1.)(6pts)Explain whether the following IVP have unique solutions?

a.) $\begin{cases} y' = \sqrt{9-x^2-y^2} \\ y(0) = 0, \end{cases}$ b.) $\begin{cases} y' = \frac{x-1}{1-y^2} \\ y(0) = 1. \end{cases}$

2.)(4pts)Solve the separable DE: $\sin x dy = y \cos x dx, \quad 0 < x < \frac{\pi}{2}$.

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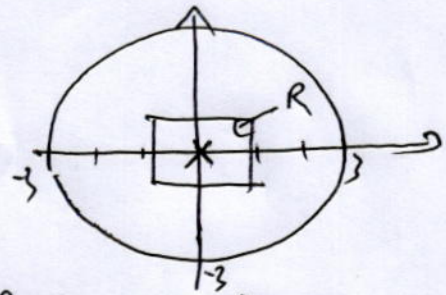
1) a) $\frac{dy}{dx} = f(x,y)$, where

$f(x,y) = \sqrt{9-x^2-y^2}$, $x^2+y^2 \le 9$

$\frac{\partial f}{\partial y}(x,y) = \frac{-2y}{2\sqrt{9-x^2-y^2}}$, $x^2+y^2 < 9$

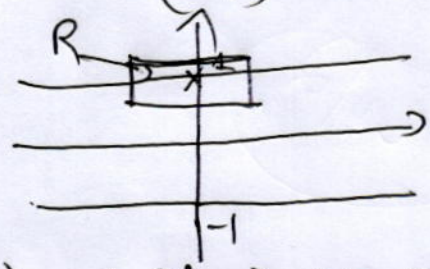


$f(x,y)$ and $\frac{\partial f}{\partial y}(x,y)$ are continuous on R; so the IVP has a unique solution y on $(-3,3)$.

b) $\frac{dy}{dx} = f(x,y)$, where

$f(x,y) = \frac{x-1}{\sqrt{1-y^2}}$, $y \neq \pm 1$

$\frac{\partial f}{\partial y}(x,y) = \frac{2y(x-1)}{(1-y^2)^2}$, $y \neq \pm 1$



$f(x,y)$ and $\frac{\partial f}{\partial y}(x,y)$ are not continuous on R. So, we can't say whether the IVP has unique solution or not.

2) $\sin x \, dy = y \cos x \, dx$, $0 < x < \pi/2$

$\int \frac{dy}{y} = \int \frac{\cos x}{\sin x} dx$

$\ln|y| = \ln|\sin x| + C$

$\boxed{y = C \sin x, \; x \in (0, \pi/2)}$